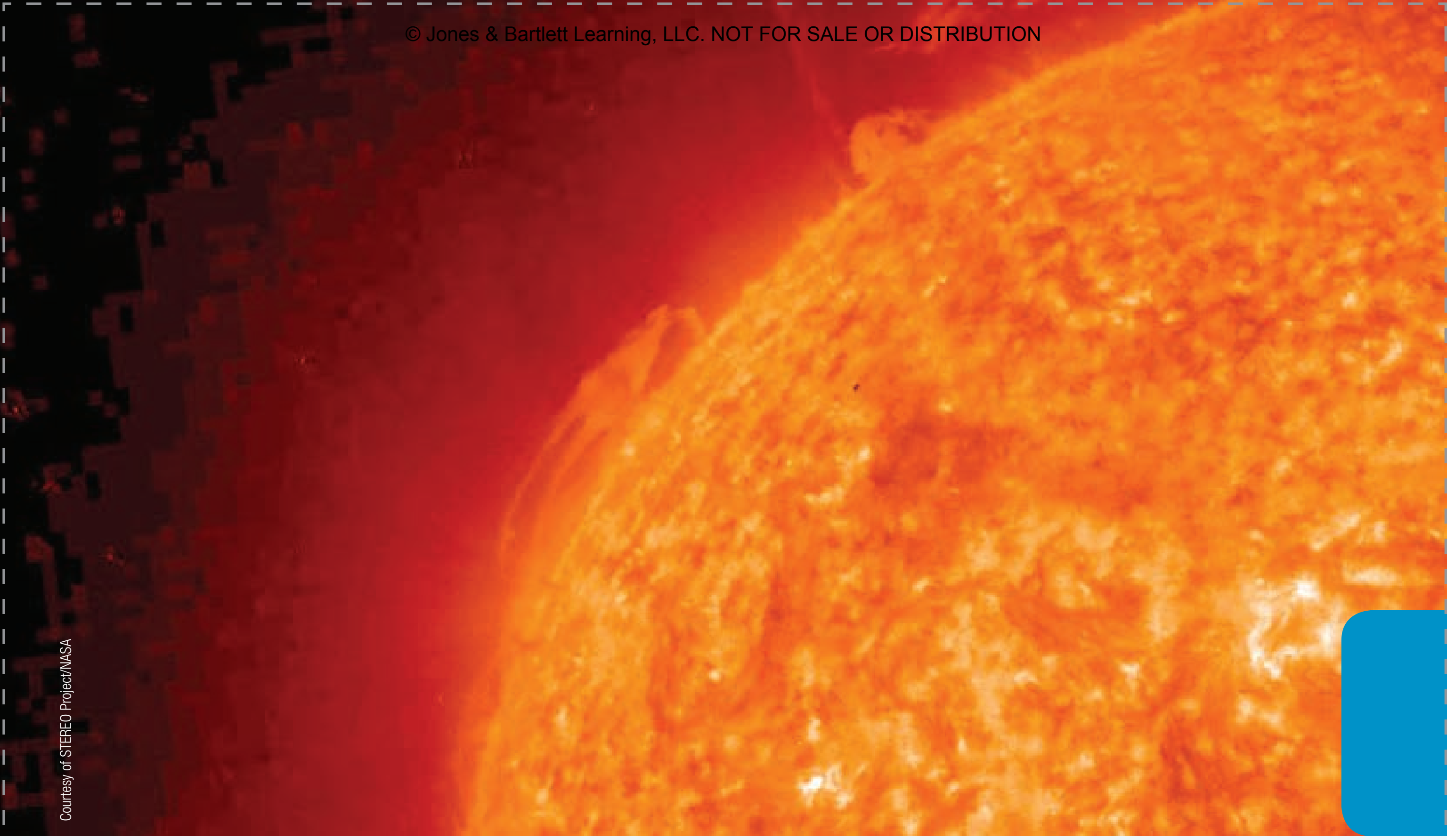

Courtesy of STEREO Project/NASA

Chapter Objectives

check off when you've completed an objective

- Know the scientific method.
- Learn units of distance and angular measure for celestial objects.
- Use the small-angle formula.
- Know the planetary systems of Ptolemy and Copernicus.
- Know and apply Kepler's three laws of planetary motion.
- Understand the concepts of velocity, acceleration, mass, weight, and force.
- Use the functions for the distance and velocity of a dropped object.
- Learn about the lives of Galileo and Isaac Newton.
- Apply the inverse-square law.
- Learn the importance of the *Principia* and the law of universal gravitation.
- Find the velocity and acceleration of a body in a circular orbit.

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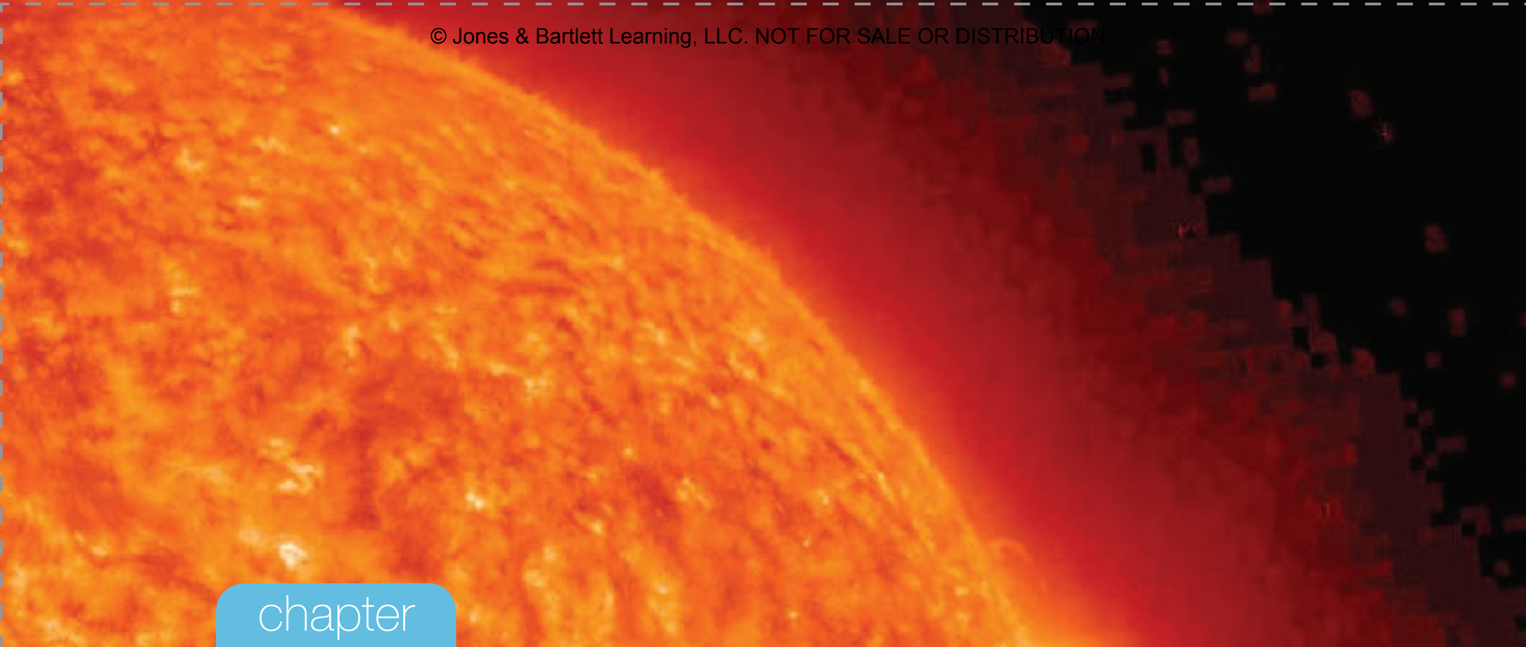

chapter

4 Astronomy and the Methods of Science

Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language. . . . It is written in the language of mathematics. . . . Without these one wanders about in a dark labyrinth.

—Galileo Galilei, *The Philosophy of the Sixteenth and Seventeenth Centuries*, translated by Richard Henry Popkin, p. 65. Copyright (c) 1966, Free Press.

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FIGURE 4.1.1 The night sky.

Few activities have enjoyed such universal appeal through the ages as gazing up at a starry sky on a pleasant summer's night (see **Figure 4.1.1**). We have all spent an idle evening wistfully musing as those faraway diamonds sparkled against the magnificent black backdrop of the cosmos. Some evenings we have taken the next step and pondered the inevitable cascade of questions that seem to hang in the air right next to the stars and planets. What are they made of? How long have they been there? How far away are they? Along what path do they move, and how fast? The search for the answers to such curiosities comprises the beautiful subject of **astronomy**. See **Figure 4.1.2**.

astronomy The science of the observation and study of the universe.

natural philosophy A set of principles or ideas concerning the workings of nature. The addition of mathematics to formalize such ideas led to the recasting of this phrase as *science*.

A brief look at the history of humankind's search to understand its place in the universe provides us with a classic example of the interweaving of the development of mathematics with the genesis of modern science and of its impact on the evolution of human culture. We shall see that mathematics is not just numbers and equations, but rather an extremely effective language for *deductive* reasoning and for defining the terms and concepts with which to debate the great questions of the world. Indeed, mathematics often has to be *created* to explain newly discovered relationships and to sort and separate facts. It has been very successful in rendering the tenets of **natural philosophy**, a phrase from an earlier era that today we would call science.

Science can be reasonably defined as systemized knowledge logically deduced from observations. Certainly a claim can be made declaring astronomy to be the *first* science; the quest to understand the celestial theater that surrounds us began several millennia ago (and continues vigorously today). This amazing odyssey provides us a rich tapestry of lives and achievements well worth our effort to examine.

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FIGURE 4.1.2 The astronomer reaches for truth. (Camille Flammarion, *L'atmosphère: météorologie populaire*, Paris, 1888.)

Replete with more failures than successes, the history of astronomy is fraught with the full range of emotions and hardships that are always the handmaidens to any human experience. Therefore, the style and tone of this chapter will be slightly different than found elsewhere in the text. A story is going to be told, and as it unfolds, we will intersperse our study of specific mathematical and astronomical principles with important events in the lives of the key contributors.

4.1 Ancient Milestones

We begin by realizing that the nocturnal scenes witnessed in the evening skies by people 2,500 years ago are pretty much the same ones you can see today in your own backyard on any clear night. Back in the days before indoor distractions such as televisions, people used to regularly gather outside to while away the evening in pleasant conversation. Attentive individuals would notice over a period of months that there were some things in the arrangement of the swarm of sparkling lights above them that stayed the same and others that changed. For example, compare the two pictures shown in **Figure 4.1.3**, giving the same view in the same direction but separated chronologically by several weeks (or possibly months).

We note that, except for the brighter object, all the stars have remained in the same locations *relative to one another*. In the right-hand picture, however, the apparent motion of the bright object through the background stars marks it as different somehow, and any model of the universe we might propose must account for the meanderings of this so-called "wanderer." Indeed, *planet* is the Greek word for wanderer, and the ancient Greeks observed five such planets (Mercury, Venus, Mars, Jupiter, and Saturn) gallivanting across the heavens.

These are observations of occurrences happening over a period of time. Likewise, no one can escape noticing the daily movements of the sun and moon rising in the east and setting in the west. Most of the stars also follow this same pattern, although if we direct our gaze to the north, we see that one particular star (Polaris) seems to not move much at all during the night while nearby stars follow circular paths around it, never dipping below the horizon. Such a stable beacon of the night has been observed for centuries both by sailors,

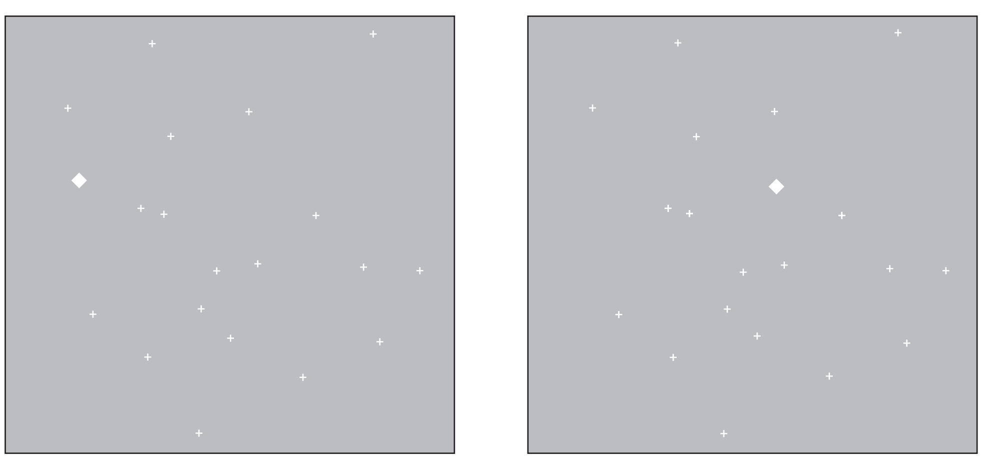

FIGURE 4.1.3 The view on the right is of the same group of stars but several weeks later.

who used it for navigating, and by bards, who sang of its permanence. In *Julius Caesar*, Shakespeare wrote:

> If I could pray to move, prayers would move me;
> But I am constant as the northern star,
> Of whose true-fix'd and resting quality
> There is no fellow in the firmament.

What model of the sun, moon, planets, and stars might you invent that would fit these facts? Incidentally, that's quite important—"fitting the facts." One key to understanding a history of the important discoveries in any scientific arena is an examination of the development and refinement of the **scientific method**, a process involving three main steps:

scientific method A formalized procedure for exploring natural phenomena involving:
1. Careful observation and the recording of data.
2. Objective analysis of the data and the creation of a model to fit the data.
3. Use of the model to make predictions that can be tested against new observations.

axioms A statement universally accepted as true.

1. **Careful observation of a phenomenon and the recording of data**
2. **Objective analysis of the data and the creation of a model to fit the data**
3. **Use of the model to make predictions that can be tested against new observations**

Inherent in the construction of any model of a natural phenomenon is the concept of the underlying *axioms*. Essentially, **axioms** are simply stated assumptions whose truth seems so self-evident that most people accept them as true without proof. Just as any sturdy edifice needs cornerstones laid as a foundation for building, so does every field of science or mathematics adopt a set of axioms as a starting point from which to proceed. The Greeks of the classical Athenian period (c. 600–350 BC) felt that such basic principles were so primordial that the knowledge of them resided within everyone, and so their realization needed no further explanation. This is akin to the notion that you need not prove the existence of your own soul—you just feel it. The problem with this approach is that the history of science and mathematics is full of instances where one or more of the foundational axioms of a particular theory have been shown to be unreliable. This can then lead to a dramatic change in the consequences of those axioms, perhaps to the point of completely abandoning the entire theory. Such a happening is called a **paradigm shift**—an altering of the standard model for a particular phenomenon; in fact, our study of astronomy will highlight one of the classic paradigm shifts in the history of science. The lesson of these revisions has led to a modern attitude in which we adopt our axioms with a bit less certainty and a bit more flexibility.

paradigm shift An altering of the standard model for a phenomenon.

We begin our exploration of the development of our current model of the planetary system with that ancient, mysterious character **Pythagoras** (c. 580–500 BC) (see **Figure 4.1.4**) and his school of followers. To the Pythagoreans, numbers were very important tools to solving the riddles of the natural world around them, an idea that grew increasingly strong as these thinkers discovered the roles that numbers played in establishing relationships in geometry and music. Furthermore, because the universe was a pure and harmonious place, they believed that these relationships extended to the movement of the planets. The "music of the spheres" was that perfect blend of tones produced by the planets moving in cosmic harmony in the same way as the proper strings of the right lengths vibrate on a musical instrument.

quadrivium A group of studies in medieval universities consisting of arithmetic, music, geometry, and astronomy.

Establishing connections between such seemingly unrelated disciplines led to the famous set of grouped studies historically known as the **quadrivium** of

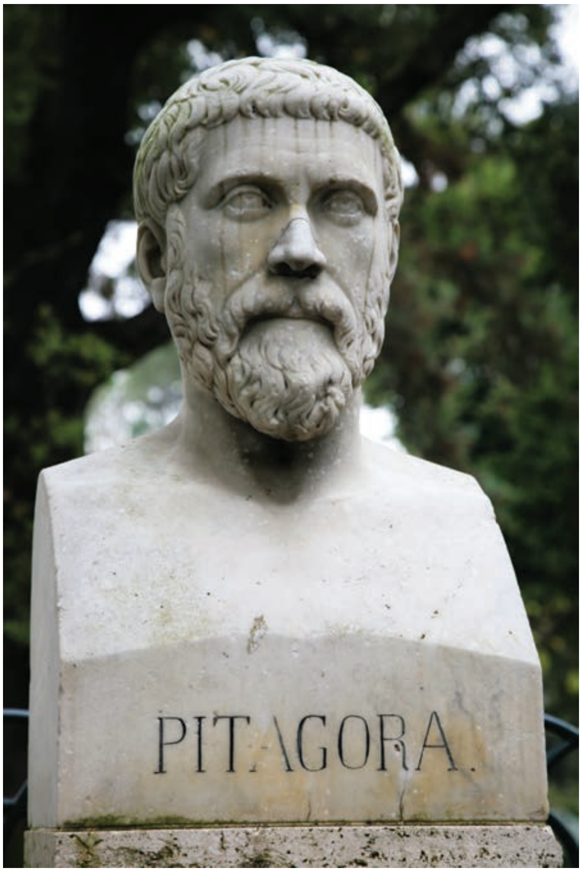

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FIGURE 4.1.4 Bust of Pythagoras.

knowledge, consisting of arithmetic, music, geometry, and astronomy. The study of these subjects led to a degree in many medieval universities, and they were all considered to be branches of the same field, mathematics.

Although the so-called music of the spheres was pure fantasy, Pythagoras and his followers also believed that because circles and spheres were the perfect forms of geometry, they must be the paths and shapes that the gods had supplied to the universe. These ideas tilled the soil for the growth of the concept of the sun, planets, and stars moving in continuous circular paths around a spherical Earth. In fact, these two primary axioms influenced most of the planetary models produced from 500 BC to the 1500s:

1. Earth is the stationary center of the universe (called a **geocentric** universe).
2. Every celestial body travels in a circular path.

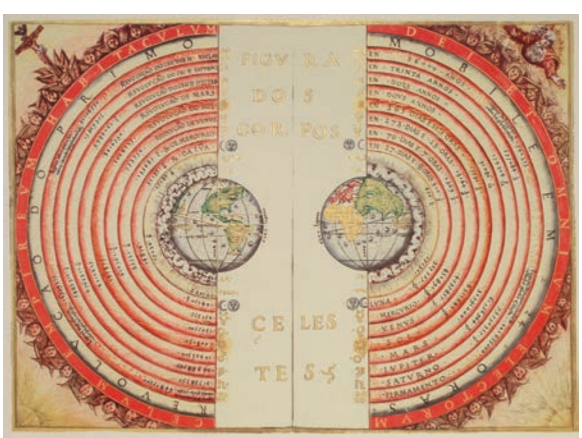
In light of the state of knowledge of the natural world at that time, such axioms are quite reasonable. Surely, thought the ancients, if Earth were sailing through empty space, wouldn't we all be sent hurtling off its surface, much like ants off a thrown apple? Circles, in fact, have had such a special place in the geometry of the world—the most graceful curve in nature—that their use remained ingrained in any depiction of the planetary orbits well into the sixteenth century, as seen in **Figure 4.1.5** in the illustration of the geocentric system done by Portuguese cosmographer Bartolomeu Velho in 1568 (Bibliothèque Nationale, Paris).

geocentric Earth-centered.

parallax The change of position of a close object with respect to a more distant background when viewed from two different locations.

Although we know today that the planets do not move in circles centered on Earth, the notions of a round Earth and planets in repeating orbits were giant steps in the right direction. Bent on using mathematics to provide answers to philosophical questions, the Pythagoreans dispensed with many previous mythologies and planted the seed for the growth of the scientific method by attempting to describe the phenomena of nature based on observation.

For example, the lack of observational *parallax* supports the notion of a stationary Earth. You already are familiar with this optical notion: Close one eye, extend your arm with your thumb held vertically, and use your other eye to line up your thumb with some object several meters distant. Now close your open eye and open the closed one. What happens? Your thumb appears to have moved with respect to the distant object. This is known as **parallax**. Well, the same should be true for our view from Earth if it is truly a vehicle that is carrying us through space. Why do the closer stars not change their orientation with respect to the more distant stars over time? The answer, of



Courtesy of the Bibliothèque nationale de France, Paris.

FIGURE 4.1.5 The geocentric system.

course, is that they do! It's just that they are *all* so far away (the closest star after the sun, Alpha Centuri, is more than 41 trillion kilometers (km) from Earth) that stellar parallax is not observable with the naked eye. See **Figure 4.1.6**.

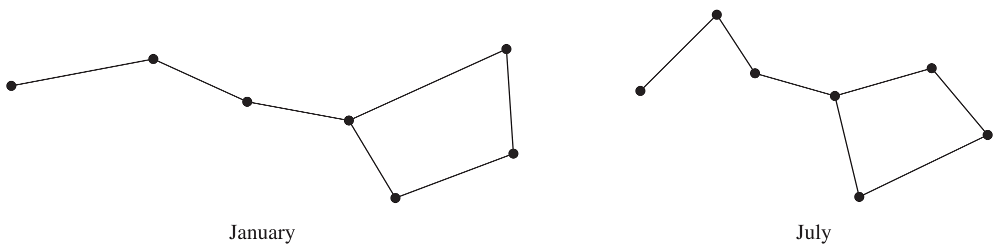


FIGURE 4.1.6 Fictitious scenario for how stars in the Big Dipper constellation might change as a result of Earth's motion if they were closer to us.

Example 1

Miles and kilometers are units of distance that are useful for terrestrial measurement but quickly become cumbersome when used to describe the vastness of space. Modern-day astronomers use the **astronomical unit** (abbreviated as AU) in computing distances among the planets, asteroids, comets, and other members of our solar system. One astronomical unit (1 AU) is equal to the mean distance of Earth from the sun—93,000,000 miles (mi) or 150,000,000 km. Incredibly, stars and galaxies are so much farther away that even astronomical units become unwieldy, and *light-years* have become the standard unit of measurement. One **light-year** (ly) is the distance light travels in 1 year (yr) at its speed of 300,000 kilometers per second (km/s). If we multiply this speed by the number of seconds in a year, we get the distance:

astronomical unit The mean distance from Earth to the sun, equal to about 93 million mi or 150 million km. Abbreviated AU.

light-year Distance traveled by light in 1 yr, equal to about 9.5 trillion km.

$$\left(300{,}000\,\frac{\text{km}}{\text{s}}\right)\left(3{,}600\,\frac{\text{s}}{\text{h}}\right)\left(24\,\frac{\text{h}}{\text{day}}\right)(365\text{ days}) \approx 9.5\times10^{12}\text{ km}.$$

So 1 ly is equal to about 9.5 trillion km. Because Alpha Centauri is 41 trillion km away, this translates to 41/9.5 ≈ 4.3 ly. ♦

Modern literature contains many references to the great distances that all celestial objects are from Earth. The following poetic passage is from *The Risk Pool* by Richard Russo.

> *Here was a wish from another lifetime, granted twenty-five years too late, as if God were in a place so distant that it took almost forever for wishes to travel there, like pale starlight from a distant galaxy, eons old and all worn out even as we look at it.*

The only reasonable way to gauge the separation between celestial objects as seen from Earth is by measuring an angle. We say, for example, that star A and star B have

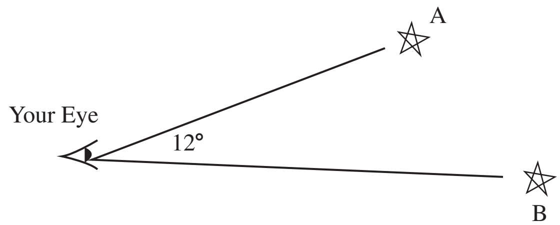


FIGURE 4.1.7 Angular separation.

angular separation
The measure of the angle formed at the observer's position by the two lines of sight to two separate objects.

arcminutes
One-sixtieth of a degree. Used as a unit for angular separation.

arcseconds
One-sixtieth of an arcminute. Used as a unit for angular separation.

an **angular separation** of 12° if the two lines of sight from the observer to those stars form an angle of 12°. See **Figure 4.1.7**. Note that this tells us nothing about the actual distance in space *between* the two stars because their separate distances from Earth could be very different. It follows that the motion of an object is often given in a certain number of degrees or radians per unit of time. Smaller units are often needed, and so 1 degree (1°) is defined to be 60 **arcminutes** (abbreviated by 60′) while 1 minute (1′) of arc is equal to 60 **arcseconds** (abbreviated by 60″).

In the following passage from *Cold Mountain* by Charles Frazier, references are made to the motion of stars in the night sky. Knowing basic astronomy can augment your enjoyment of literature.

> Orion had fully risen and stood at the eastern horizon, and from that Inman made the time to be long past midnight. Orion was girded about tight, his weapon ready to strike. Traveling due west every night and making unfailing good time.

Example 2

As the sun moves across the sky each day, you could easily compute that it moves at a rate of 15° per hour. The ancients thought the sun was circling Earth, but of course today we know that Earth rotates on its axis once every 24 h (360° divided by 24 h yields the required rate). Naturally, the stars at night display this same apparent motion and at the same angular rate, but you will notice that the path each individual star appears to follow is actually along a circle centered at Polaris, which itself never moves. This is so because the northern end of our planet's rotational axis points (almost) directly at Polaris (the end star in the handle of the Little Dipper). Stars in the northern part of the sky, such as those in the constellations Cassiopeia and the Big Dipper, move in tighter circles. Stars that have a greater angular separation from Polaris, such as those in Orion, move in circles large enough to give the appearance of rising in the east and setting in the west. (In the southern hemisphere, an analogous arrangement prevails with circles of apparent motion centered at a spot in the southern part of the sky.) ♦

Example 3

If you were to look directly east at whatever group of stars is peeking over the horizon every night at suppertime, you would easily ascertain after a few weeks that those stars were appearing about 1° higher each night. (Looking at the stars at the same time every night eliminates the problem of accounting for the daily rotation of Earth.) Six months from now, that same group is setting in the west at suppertime, and in a year's time, it has returned to its original position. This type of motion is physically distinct from that described in Example 2, but as in that example, the movements are along Polaris-centered circles. Earth's annual revolution around the sun accounts for this transport through 360° over a period of 365 days, yielding a star "marching speed" of about 1° per day toward the west. Careful measurement of any constellation would show a movement of about 30° over a 1-month period. ♦

As we mentioned earlier, the mysterious travels of the five visible planets had inspired the wonder of habitual stargazers for centuries. Why do these five orbs appear to move among the background of an ocean of stars? Well, for one thing, they must be much closer than the stars. Moreover, there was little argument as to their relative proximities because of the assumption that the more slowly the planet moved, the farther it must be from Earth. So observations of the planets' relative speeds put them in the order of Mercury, Venus, Mars, Jupiter, and Saturn.

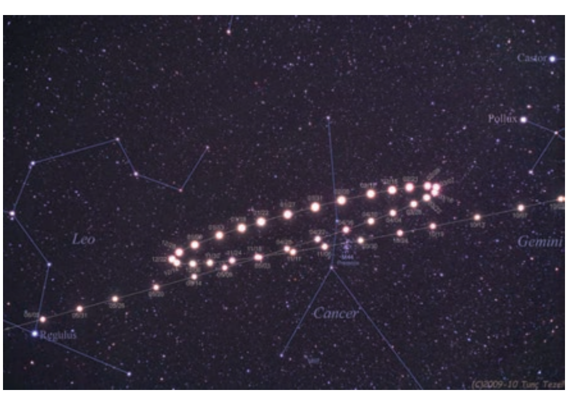
However, two major problems could not be explained by simple circular motion around the Earth. The motions of all the planets are restricted to a rather narrow ribbon of the sky called the **zodiac** that also contains the apparent path of the sun. (We know today that this is the result of all the planets orbiting the sun in pretty much the same plane.) Although Mars, Jupiter, and Saturn typically travel in eastward paths through the zodiac, one of them occasionally slows to a complete stop, reverses its motion, and goes westward for a few weeks or months until it stops and reverses again to resume its original direction (**Figure 4.1.8**). This reversal is referred to as **retrograde motion**, and it was a mystery that all the early cosmologists struggled to resolve. Accounting for retrograde motion under the axiom system described earlier eventually led to the **Ptolemaic system**, a model we will examine closely in the next section.

zodiac A narrow, beltlike region around the sky containing the apparent paths of the sun and planets.

retrograde motion The apparent reversal of the movement of a planet.

Ptolemaic system Model of the planetary system with Earth at the center and employing epicycles and deferents to explain retrograde motion of the planets.

The other big question concerned the fact that something altogether different was going on with Mercury and Venus. These two planets could be seen only in the early morning just before sunrise or in the evening just after sunset, but never in the middle of the night. They stuck close to the sun with Venus never separated by more than 48° and Mercury never by more than 28°. It was this riddle that **Heracleides** (c. 388 BC) solved by being the first to suggest the incredibly novel idea of having Mercury and Venus in orbit *around the sun*. (See **Figure 4.1.9**.)



Courtesy of Tunç Tezel

FIGURE 4.1.8 The position of Mars taken at 10-day intervals.

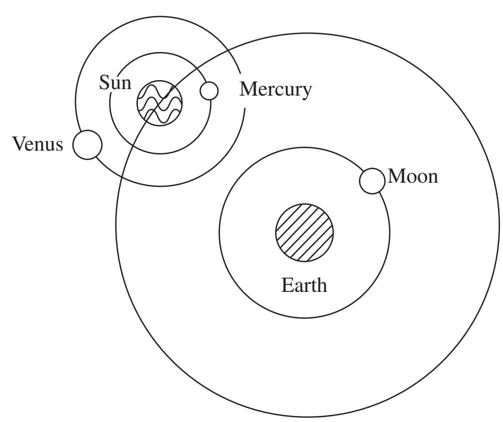


FIGURE 4.1.9 The model of Heracleides.

Now here was a thought of far-reaching significance indeed! True, he still had the sun circling the Earth, but no one had ever offered up the possibility of anything moving around the *sun*. We now use this revised picture to compute some relative distances.

Although undetectable by the naked eye, the width of the planetary disks varies as their distance from us changes. Modern telescopes can determine the angular width of the visible disk of a celestial body with great precision. The **angular diameter** α of any object (as opposed to its actual or linear diameter) is defined as the angular separation from one side of the object to the other side. By measuring the angular diameter, you can find the distance to the object if you know the linear diameter, or vice versa. The distance to most celestial objects is great enough to allow us to think of the linear diameter pictured in **Figure 4.1.10** as a portion of a large circle whose radius is the distance. If we use radians to measure α, then geometry tells us that

angular diameter The angular separation of opposite sides of an observed object.

$$\alpha = \frac{\text{linear diameter}}{\text{distance}}.$$

However, α is generally measured more conveniently in arcseconds because of its small size, and so we may utilize the fact that 1 radian (rad) consists of 206,265 arcseconds to obtain the proportion

$$\frac{\alpha}{206{,}265} = \frac{\text{linear diameter}}{\text{distance}},$$

where α is now measured in arcseconds. The above relationship is often referred to by astronomers as the *small-angle formula*.

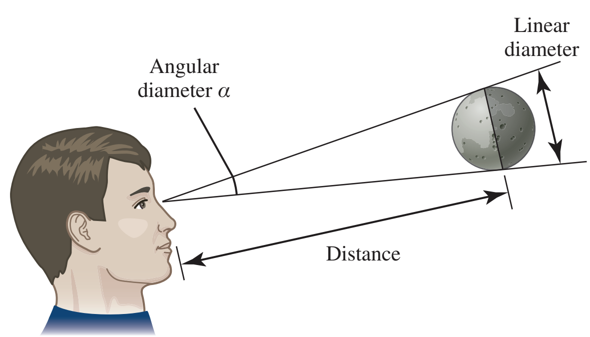


FIGURE 4.1.10 Angular and linear diameters.

Example 4

The moon's diameter is 3,500 km. We can sometimes assume a constant Earth–moon distance, even though, in reality, it is continuously changing. If the distance varies in one month from a minimum of 363,000 km to a maximum of 405,000 km, what are its minimum and maximum angular diameters that month?

Solution

Substituting the appropriate numbers from above into the small-angle formula, we get

$$\frac{\alpha}{206{,}265} = \frac{3{,}500}{363{,}000},$$

and this gives us

$$\begin{aligned}\alpha &= 0.00964(206{,}265)\\ &= 1{,}990''\\ &= 33'\end{aligned}$$

for the maximum angular diameter. Likewise,

$$\begin{aligned}\frac{\alpha}{206{,}265} &= \frac{3{,}500}{405{,}000}\\ \alpha &= 0.00864(206{,}265)\\ &= 1{,}780''\\ &= 30' \text{ for the minimum.}\end{aligned}$$

Although this fluctuation of 3′ is probably not noticeable to the casual observer, it is apparent in the differences it causes in total solar eclipses. At maximum distance, the smaller lunar disk fails to cover the entire sun, producing a thin brilliant ring of fire surrounding the disk. It is known as an *annular eclipse*. ♦

The Egyptian city of Alexandria, established by the warrior-king Alexander the Great in 322 BC, was the cultural and intellectual capital of the western world for several hundred years. One of the giants of this era was unquestionably the brilliant **Aristarchus** of Samos, who was born around 310 BC. Although only a single book of his writings survived to the present—*On the Sizes and Distances of the Sun and the Moon*—his fame is secured by the respect he was accorded in the scientific literature produced by his contemporaries. Aristarchus combined sharp mathematical reasoning with keen fact gathering to formulate answers to difficult questions.

Example 5

Aristarchus invented an ingenious method for determining the ratio of the Earth–sun distance to the Earth–moon distance and, in the process, greatly enlarged the Greek estimates of the size of the universe. His technique rested on examining the right triangle formed by the sun, moon, and Earth when the moon is at the position in its orbit that yields the view of a half-illuminated disk (first-quarter or third-quarter phase).

Somehow Aristarchus estimated the angle θ in **Figure 4.1.11** to be about 87°, and this then implies that the small angle must be 3°. Because each degree equals 3,600″, the small-angle formula can be applied to give us

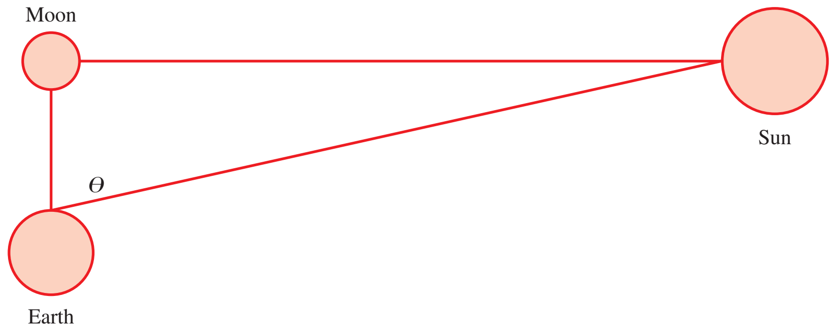


FIGURE 4.1.11 The method used by Aristarchus to compare the distances to the sun and moon.

$$\frac{10{,}800}{206{,}265} = \frac{\text{EM}}{\text{ES}},$$

where EM = the Earth–moon distance and ES = Earth–sun distance. Therefore,

$$\begin{aligned} 0.0524 &= \frac{\text{EM}}{\text{ES}} \\ \text{ES} &= \frac{\text{EM}}{0.0524} \\ &= 19\,\text{EM}. \end{aligned}$$

Note that a small change in the measurement of the angle induces a large change in this factor (see the exercises). Although the actual ratio of the Earth–sun distance to the Earth–moon distance is about 390, this was nonetheless a truly astounding discovery for that era. ♦

Among his many accomplishments, Aristarchus is commonly credited with being the first man to place *the sun at the center* of the planetary system with *all* the planets, including Earth, in circular orbits around it. This is known as a **heliocentric** system (the Greek word *Helios* means sun god). He felt the scale of the universe to be so grand and the stars so enormously distant that any parallax caused by the movement of our planet would not be measurable. These ideas gained little acceptance in Aristarchus's lifetime. Perhaps Aristarchus himself was not fully convinced of the truth of his conjecture because there existed no strong observational evidence to favor this scheme over any other. It was simply a philosophical alternative that seemed, somehow, less secure than that of a geocentric universe. In fact, it would be almost 1,800 years before a sun-centered planetary model would be seriously considered again.

heliocentric
Sun-centered.

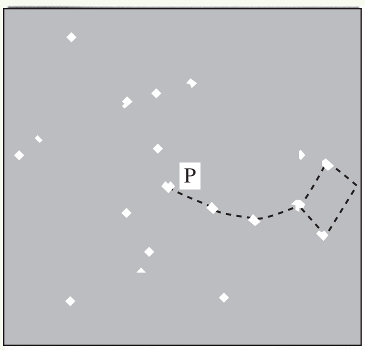
The scanty conceptions to which we can attain of celestial things give us, from their excellence, more pleasure than all our knowledge of the world in which we live.

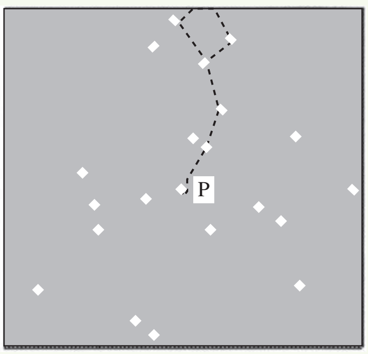
—Aristotle (*Parts of Animals* I, 5)

Name ______________________

Exercise Set 4.1

1. These figures show two simulated views of the same area of the sky taken 6 h apart. The star labeled P is Polaris, commonly called the North Star because the North Pole of Earth's rotational axis points almost directly at it. Polaris is the end star in the handle of the Little Dipper constellation, which has been sketched in for easy reference. Note how the location of Polaris is unchanged in the second picture, while the rest of the stars in the constellation *appear* to have rotated around Polaris in circular paths. What phenomenon regarding the daily motion of Earth actually accounts for this observation? The entire constellation has rotated counterclockwise about 90°. Thus, we see that each star moves at an angular rate of 90° every 6 h, or 15° per hour. Is this consistent with your answer to the above question? Why?





2. Suppose you go out tonight at 9:00 P.M. and pick out a nice bright star on the eastern horizon. If you view that same star 1 month from now at 9:00 P.M., what angular displacement (on a large circle centered at Polaris) will that star have undergone?

3. Explain why the assumption of a stationary Earth was so prevalent for such a long time.

4. What observational facts led Heracleides to propose that Mercury and Venus orbited the sun instead of Earth?

5. Name the two axioms for building any planetary model that were believed by most of the Greek astronomers of antiquity.

6. How many degrees does the sun move across the sky between 10:00 A.M. and 3:00 P.M. due to the rotation of the Earth?

7. What is the angular rate at which the stars rotate around Polaris in units of degrees per hour? Arcminutes per minute? Arcseconds per second?

8. One *light-second* is the distance light travels in 1 s. What is the mean distance of Earth from the sun in light-seconds? In light-minutes? [Recall that the speed of light is 3×10^5 km/s and the distance from Earth to the sun is 1.5×10^8 km.]

9. The mean distance of Pluto from the sun is about 39 AU. What is that distance in light-seconds?

10. How many astronomical units are in 1 ly?

11. The moon takes roughly 30 days to complete one trip around Earth. Compute the angular rate (in degrees per hour) at which the moon moves through its orbital path.

12. Phobos, one of the two moons of Mars, orbits the planet at a distance of about 5,980 km. It is considerably smaller than our own moon, having a diameter of only 20 km. What angular diameter would Phobos present to an observer on Mars? Would Phobos appear larger or smaller in the Martian sky than our moon appears to us here on Earth?

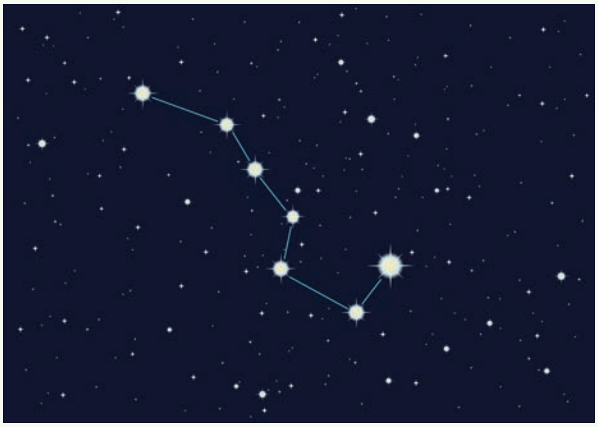
13. The maximum angular diameter of Jupiter as seen from Earth is about 50″ (arcseconds). If the linear diameter of Jupiter is 144,000 km, determine the minimum distance (in kilometers) of Jupiter from Earth.

14. The planet Saturn is well known for its colorful rings. The diameter across the outermost ring is 340,000 km. When Saturn is 9.0 AU from Earth, what is the angular diameter of the outermost ring?

15. In the previous exercise, what is the angular diameter of the outermost ring of Saturn when the planet is 10.0 AU from Earth?

16. If d and α are the linear and angular diameters, respectively, of a celestial object and r is the distance to the object, then, for any particular object (cf. Example 5), α clearly changes as r changes. Write α as a function of r. Does α achieve a maximum or a minimum when r is a minimum? Does α achieve a maximum or a minimum when r is a maximum?

17. If the angular separation between two objects is less than 5°, for our purposes we may continue to assume that the triangle formed by those two objects and Earth is a long, narrow right triangle. Megrez and Phad are two of the stars forming the bowl of the constellation commonly called the Big Dipper. Both of these stars happen to be about 80 ly from Earth and have an angular separation of 4.5°. How far apart are they in space? (1° = 3,600″.)



18. Many of the points of light in the night sky that appear to be single stars are actually composed of two stars revolving around a common point between them. These are called *binary systems*. A binary system appears to be a single star because it is too far away for the human eye to resolve into two separate light sources. About the closest star separation that the eye can distinguish is 4′. Mizar, a binary system in the crook of the handle of the Big Dipper (see above picture), is about 73 ly from Earth and can only be resolved by the keenest human eye. What must be the maximum distance between the two stars this system?

19. Alpha Centauri, our sun's closest stellar neighbor at 4.3 ly, is actually a triple star system. The two biggest, brightest stars are known as Alpha Centauri A and B. They orbit each other every 80 yr, having minimum and maximum separations of 11 and 35 AU, respectively. What are the corresponding minimum and maximum angular separations of these two stars as seen from Earth? Would you need a telescope to identify these as two different stars? (From the previous exercise, we see that the answer is yes if the angular separation is always less than 4′.)

20. Aristarchus computed a distance to the moon of about 80 Earth radii (the actual figure is closer to 60) by using an ingenious technique that involved clocking the time it took the moon to pass through Earth's shadow during a lunar eclipse. Refer to Example 5 to determine what Aristarchus concluded must be the distance to the sun.

21. A more accurate measurement of the sun–Earth–moon angle θ in the Aristarchus triangle in Figure 4.1.11 is 89.853°. If the mean distance to the moon is about 384,000 km, what is the mean distance to the sun?

22. A *solar eclipse* occurs when the moon interposes itself between the sun and Earth. It is a dynamic demonstration that the moon is closer to us than the sun and also reveals that the average angular diameters of these two bodies are the same—about 30′. Similar triangles then allow us to conclude that the *ratio* of the diameters of the sun and moon is the same as the *ratio* of their distances from Earth:

$$\frac{\text{Distance to sun}}{\text{Distance to moon}} = \frac{\text{diameter of sun}}{\text{diameter of moon}}.$$

The work of Aristarchus in Example 5 gave a value of 19 for the ratio on the left-hand side of the equation in the figure. If the diameter of the moon is given to be 3,500 km, what would be the diameter of the sun?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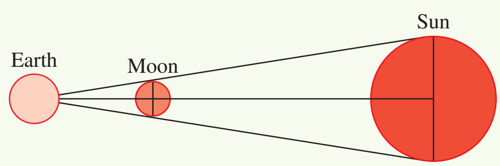
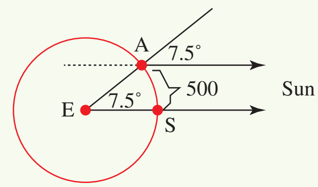


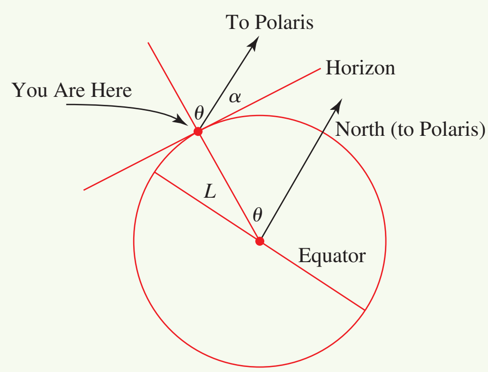
23. In Exercise 21, you computed a different value for the ratio of the Earth–sun distance to the Earth–moon distance than the one computed by Aristarchus. Use this value to answer the same question as in Exercise 22: If the diameter of the moon is given to be 3,500 km, what is the diameter of the sun?

24. One of the greatest scholars of the Alexandrian Greek world was **Eratosthenes** (275–194 BC). He was the first to compute an extremely good estimate of the circumference of Earth. Eratosthenes noticed that at summer solstice, the sun was directly overhead in the city of Syene (S in the accompanying picture) and at an angle of 7.5° from straight overhead in Alexandria (A), which was located 500 mi to the north. The angle at the center E of Earth is formed by radii extended through both points. Because we can think of sunlight rays as parallel, the measure of this angle must also be 7.5°. Because $\frac{7.5}{360} = \frac{1}{48}$, this means that the portion of the circle from A to S is $\frac{1}{48}$ of the circumference. What is the circumference?



25. Suppose Eratosthenes measured the angle at E in the accompanying picture to be 6°. What would have been his value for the circumference of Earth?

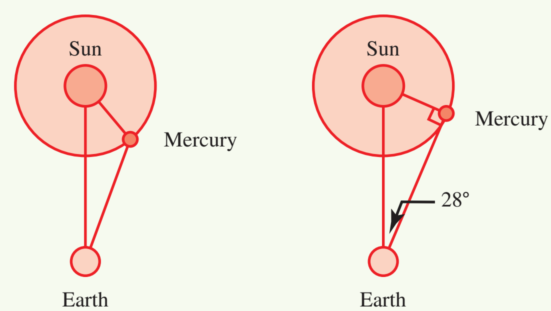
26. If you were to travel to an exotic land and wished to locate the North Star some evening, there is a wonderful little fact you should know. The elevation angle from the horizon to Polaris is related to the latitude of your location. For instance, where would Polaris be in the sky if you were situated at the North Pole? At the equator? Your *latitude angle L* is defined to be the angle between radii drawn to the equator and to your location. In the picture here, your line of sight to Polaris and the line from Earth's center to the North Pole are parallel. Therefore, the two angles marked by θ are equal. Prove that the angle of elevation α is equal to L.



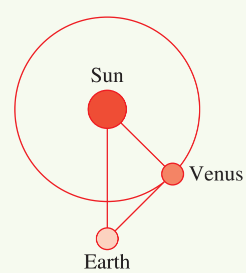
27. In the previous exercise, would Polaris appear higher in the sky as viewed from Dallas, Texas, or Chicago, Illinois?

28. **Elongation** of a planet is the angular separation of the planet from the sun as seen from Earth. The angle *SEM* in the figure here is the angle of elongation of the planet Mercury for two different positions of Mercury in its orbit. The maximum elongation angle for Mercury is about 28°, giving the configuration in the diagram on the right. We see that the line *EM* is tangent to the orbit of Mercury and therefore is perpendicular to the radius *MS* of the orbit. Hence, the three bodies form a right triangle with hypotenuse equal to the Earth–sun distance *ES*. Estimate the distance from Mercury to the sun and the minimum distance to Mercury from Earth in astronomical units. (Trigonometry is needed.)

elongation The angular separation between the sun and a planet.



29. The maximum elongation angle for Venus is about 48°. As in the previous exercise, find the distance from Venus to the sun in both astronomical units and kilometers. At this elongation, how far apart are Earth and Venus? If the linear diameter of Venus is about 12,000 km, what would be its angular diameter at this point in its orbit?



30. Use the results of the previous exercise to find the minimum distance Venus can ever be from Earth. Give your answer in both astronomical units and kilometers. If the linear diameter of Venus is about 12,000 km, what would be its angular diameter at this point in its orbit?

4.2 The Two Great Systems

> To be accepted as a paradigm, a theory must seem better than its competitors, but it need not, and in fact never does, explain all the facts with which it can be confronted.
>
> —Thomas S. Kuhn

The creative thinking of people like Heracleides and Aristarchus drives the progression of science, but a recurring problem in those olden days was the lack of sophisticated instrumentation to provide the data to test a new model. Without hard numerical data to corroborate a proposed scheme, debates about the best model of the universe continued unabated, and the set of axioms used to build any theory played a large role. Of course, in the fourth century BC, one large barrier to real knowledge was a lack of physical concepts that could be measured. Notions such as weight, mass, force, velocity, acceleration, and so on were either partially or wholly undeveloped quantitatively. One might speak of heavy or light, but never of 175 pounds (lb). In such an environment, the people who became revered authorities of learning were those who could articulate their philosophies about the nature of the world with the best rhetorical polish. Chief among these were **Plato** (429–348 BC) and his esteemed pupil **Aristotle** (384–322 BC), a man whose clarity of insight empowered his writings on philosophy, economics, history, politics, poetry, drama, and the sciences (see **Figure 4.2.1**). His main work detailing his thoughts about the structure of the cosmos is *On the Heavens*. The high esteem awarded to the circle and sphere is evident in this book. He wrote:

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FIGURE 4.2.1 Aristotle on a 5 drachma coin.

> Let us consider generally which shape is primary among planes and solids alike. Every plane figure must be either rectilinear or curvilinear. Now the rectilinear is bounded by more than one line, the curvilinear by one only. But since in any kind the one is naturally prior to the many and the simple to the complex, the circle will be the first of plane figures. . . . And the sphere holds the same position among the solids. For it alone is embraced by a single surface, while rectilinear solids have several. The sphere is among solids what the circle is among plane figures. . . . The shape of the heaven is of necessity spherical; for that is the shape most appropriate to its substance and also by nature primary.

We see these arguments as favoring not only the axiom of circular motion, but also the axiom of a spherical Earth. In fact, in a departure from the approach of Plato, who disdained experimentation, Aristotle included some observations in his studies on this point, stating:

The evidence of the senses further corroborates this. How else would eclipses of the moon show segments shaped as we see them? . . . In eclipses the outline is always curved: and, since it is the interposition of the earth that makes the eclipse, the form of this line will be caused by the form of the earth's surface, which is therefore spherical. Indeed there are some stars seen in Egypt and in the neighborhood of Cyprus which are not seen in the northerly regions . . . all of which goes to show not only that the earth is circular in shape, but also that it is a sphere of no great size. . . .

See **Figure 4.2.2**.

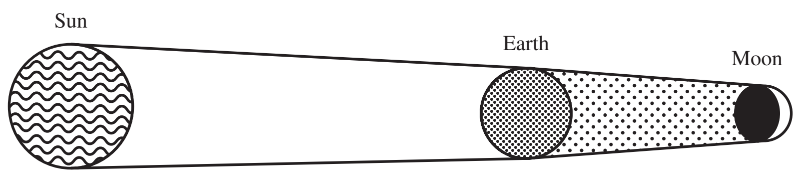

FIGURE 4.2.2 Observations of lunar eclipses led Aristotle to believe Earth was a sphere.

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FIGURE 4.2.3 A medieval painting of Ptolemy.

Plato was convinced that God had designed a perfect mathematical pattern for the motions of all the celestial bodies. Aristotle's view of the planetary model was one that wholeheartedly endorsed Platonic geocentricity but labored to explain retrograde motion. Aristotle elaborated on a model proposed by the mathematician **Eudoxus** (408–355 BC)—one that accounted for the strange reversals of the planets by having them attached to a complicated nest of interlocking ethereal spheres, each turning around Earth at a different yet uniform speed. These teachings of Aristotle led the mainstream of Greek astronomy to later embrace a geocentric, or Earth-centered, model that was to become known as the Ptolemaic system. See **Figure 4.2.3**.

Preserving both geocentricity and circular motion while simultaneously accounting for observed retrograde was accomplished through a mathematical creation inspired originally by **Hipparchus** (c. 150 BC). His idea allowed for an object in orbit around Earth to follow a path that was a geometric combination of two circles and still yielded observations of apparent back-and-forth motion. Further refinements and applications of this scheme to all the planets were made 300 years later by the last of the renowned Alexandrian astronomers, **Claudius Ptolemy** (c. 150 AD), for whom the entire model is named. The essential element is that each planet P travels in a small circle (see **Figure 4.2.4(a)**) called an **epicycle** at the same time as the center Q of the epicycle moves along another larger (imaginary) circle surrounding Earth called a **deferent**. Imagine a small ball (the planet) attached to a string 1 foot (ft) long that is tied to the end of a 5-ft stick, which you are holding. If you (Earth) turn in place while you gyrate the stick so as to twirl the ball in the same plane as the stick, the resulting motion of the ball is what Hipparchus had in mind. It moves in a curve that mathematicians call a *cycloid*, shown in **Figure 4.2.4(b)**.

epicycle Imaginary circle around which a planet moved in the Ptolemaic system. The center of the epicycle moved along the deferent.

deferent Imaginary circle centered on Earth, invented by Hipparchus, around which the center of a planet's epicycle moved.

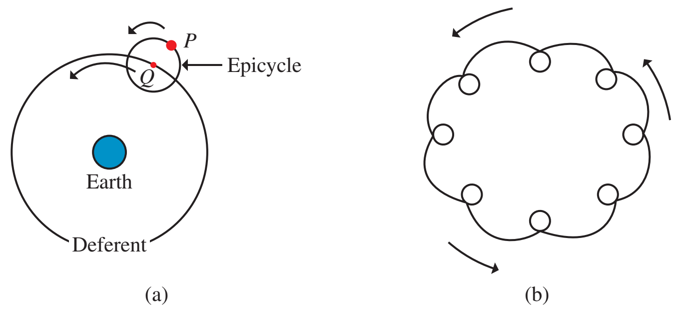

FIGURE 4.2.4 An epicycle combined with a deferent gives a geocentric explanation of retrograde motion.

Although the complete plan of Ptolemy contained some other modifications to this basic framework, note how beautifully it accounts for the retrograde problem. Each loop in the cycloid is viewed as retrograde motion from Earth. It is produced by choosing a speed for *P* greater than that for *Q*, along with appropriate radii for the epicycle and deferent. By tinkering with the sizes and angular rates of the pair of circles associated with a particular planet, you could predict its position with enough accuracy to match the observations available in that era.

The entire model and the accompanying mathematics are contained in Ptolemy's *Almagest*, one of the most significant books of antiquity. In the final configuration (**Figure 4.2.5**), Ptolemy ignored the suggestion of Heracleides concerning Mercury and Venus (as Aristotle had) and placed the centers of the epicycles containing these two bodies on an imaginary line connecting Earth to the sun, whose own orbit was placed beyond that of Venus. This very neatly explained the perpetual proximity of the two innermost planets to the sun within the constructs of the model.

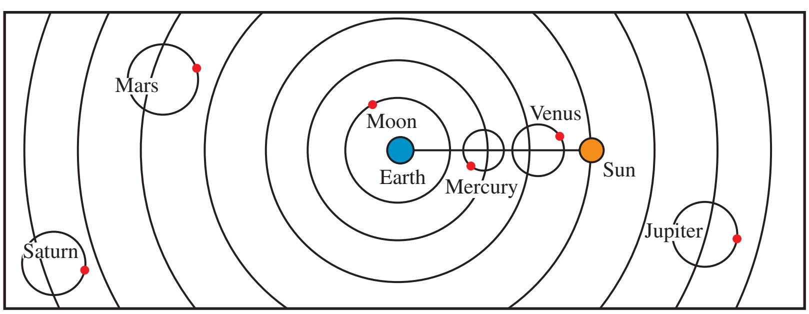

FIGURE 4.2.5 The Ptolemaic system.

So here we have a geocentric model of the universe in which all bodies moved in circles yet gave fairly good matches of their positions with observational data. It was an achievement of monumental proportions. If the primary criterion by which to judge the success of a model is the duration of acceptance by the learned community, then the Ptolemaic system—which remained the standard for more than 1,500 years—is among the most successful in the history of science. Morris Kline, one of the most respected mathematics writers of the twentieth century, called the Ptolemaic system the "supreme achievement of all Greek efforts" and wrote in his book *Mathematics and the Search for Knowledge* that

No other product of the entire Greek era rivals the *Almagest* in the profound influence it exerted on the conceptions of the universe.

During the next 15 centuries, the development of new mathematics in the West and the associated inquiries of natural philosophy came to a stop. The penetration to Mediterranean lands by fierce barbarian tribes from the north, culminating in the sack of Rome in 410 AD, began a lengthy period of chaos and suffering. The teachings of Aristotle were considered the foundations of education, and because religious doctrine also endorsed a geocentric view of the universe, no serious disagreement with the Ptolemaic system emerged for quite some time.

Then a series of events in the late medieval period began to create a new atmosphere for learning. The compass; the printing press; easier access to books; lenses, corrective glasses, and improved vision; and later the invention of the microscope and telescope all fueled an invigorating spirit of investigation. A new guiding principle of observation combined with reason began to replace a reliance on authority. As the accuracy of the data increased, the errors inherent in the geocentric model became more noticeable and prompted a reexamination of the grand system of Ptolemy.

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FIGURE 4.2.6 Nicholas Copernicus. This 1973 stamp was issued in honor of the 500th anniversary of his birth.

Nicholas Copernicus (1473–1543) (see **Figure 4.2.6**) never had any intention of discrediting the astronomy of Ptolemy, for whom he had the utmost respect. Born in Poland, Copernicus was adopted and raised by a wealthy and powerful uncle, a bishop who valued science and education as highly as religion. He sent his nephew to study in Italy, where the young man probably first hatched his seminal ideas that led to a new view of the cosmos. However, cautious by nature and fearful of condemnation by the Church as well as scorn from contemporary scientists, Copernicus did not publish his classic astronomical work, *De Revolutionibus Orbium Caelestium* (meaning *On the Revolutions of the Celestial Orbs*), until just prior to his death. (It is interesting to note that this book led to the meaning of *revolutionary* as being radically new.)

The two main claims of his treatise were that Earth was just one of six planets in circular orbit around a stationary sun (**Figure 4.2.7**) and that the daily passages of the sun, moon, and stars across the sky were an illusion created by Earth's rotation on its axis. *De Revolutionibus* began the reevaluation process that eventually dethroned the Ptolemaic system. In particular, it lent such momentum to the heliocentric model of the planets that the entire theory is often referred to as the **Copernican system**. It is unknown whether Copernicus obtained his ideas from Aristarchus of ancient Greece, but in the preliminary version of his manuscript, he wrote:

Copernican system Model of the planetary system with the sun at the center and Earth rotating on its own axis.

> Philolaus believed in the mobility of the Earth, and some even say that Aristarchus of Samos was of that opinion.

We saw how to estimate the solar distances of Mercury and Venus in the last section. Copernicus devised a clever geometric technique to also compute the distances (relative to Earth's distance) of the planets lying beyond Earth's orbit. These distances are given in

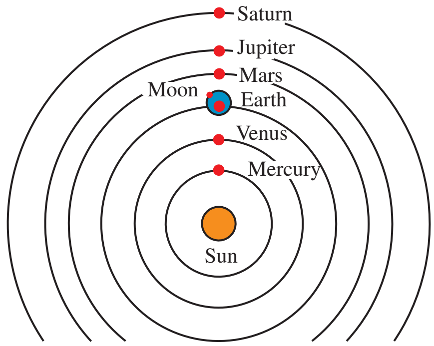


FIGURE 4.2.7 The Copernican system.

Figure 4.2.8, along with the modern astronomical unit value and the orbital period (time of 1 revolution around the sun).

Planet	Copernicus (AU)	Modern (AU)	Period (yr)
Mercury	0.38	0.387	0.24
Venus	0.72	0.723	0.62
Earth	1.00	1.00	1.00
Mars	1.52	1.52	1.88
Jupiter	5.22	5.20	11.87
Saturn	9.17	9.54	29.46

FIGURE 4.2.8 Distances and periods of the planets known at the time of Copernicus.

As we mentioned earlier, even in ancient times, the order of the planets was well established and was based on the assumption that the more distant the planet, the slower it moved. By computing each planet's solar distance, Copernicus was now in a position to estimate its velocity.

Example 1

If we suppose each planet to move at a constant rate in a perfect circle of radius a centered at the sun, then it travels a distance $2\pi a$ (the circumference of the circle) during one period p. Therefore, its velocity must be $v = \frac{2\pi a}{p}$ AU/yr . These units are not very meaningful to us because we are more accustomed to terrestrial terms, and so we convert it to kilometers per second (km/s). There are 1.5×10^8 km in 1 AU and $365\times24\times60\times60 = 3.1536\times10^7$ s in 1 year. So, 1 AU/yr $= \frac{1.5\times10^8}{3.1536\times10^7}$ km/s ≈ 4.76 km/s.

Multiplying the above formula by 4.76 gives

$$v = \frac{9.52\pi a}{p}.$$

For instance, the orbital speed of Mercury would be $v = \frac{9.52\pi(0.38)}{0.24} \approx 47.4$ km/s.

Copernican velocity Speed of a planet assuming it traveled in a circular orbit.

Today, we know the planets move along near-circular orbits at nonconstant speeds. Hence, this is only a good estimate of the average speed of Mercury, which we shall call the **Copernican velocity**. ♦

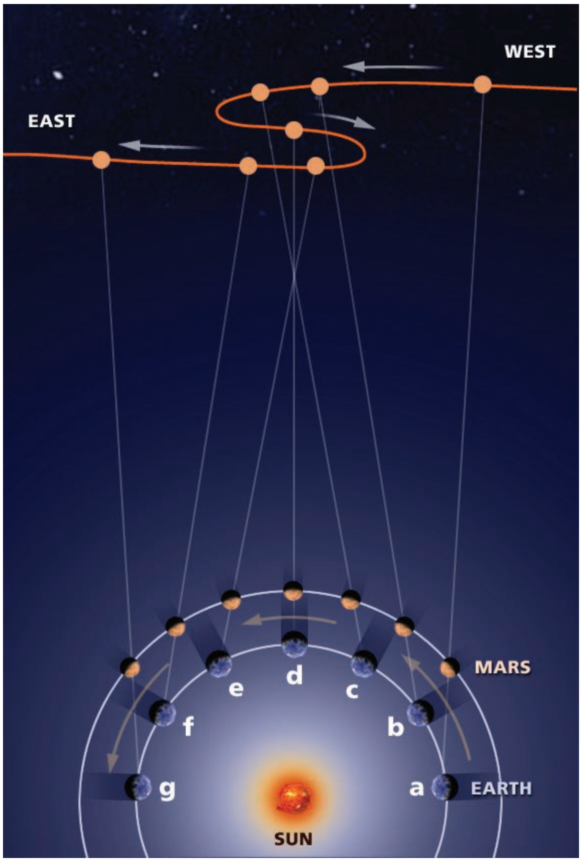


FIGURE 4.2.9 Retrograde motion explained by the Copernican system.

What prompted Copernicus to examine the time-honored Ptolemaic system? One primary reason was that putting Earth in motion around an immobile sun provided a much simpler and more elegant explanation of retrograde motion. Imagine two horses running at the same speed on a racetrack. Although they may be racing dead-even on the straightaway, the one on the inside lane will edge ahead on the curve. To the jockey on the inside horse, the outside horse appears to be going backward in spite of its forward motion. Now consider the six known planets as the horses, with Earth in the third lane from the inside. In the heliocentric system, this phenomenon, in combination with the different speeds of the planets, produces the optical illusion of retrograde motion. **Figure 4.2.9** displays the geometry of the situation for the planet Mars. Note that the reversals do not occur along exactly the same path, resulting in an S shape. This is so because the planetary orbits lie in slightly different planes.

Recall that the Ptolemaic system had been the *paradigm*, or standard model, of the planets for 1½ millennia. By replacing the first axiom (geocentricity) with a new one (heliocentricity), Copernicus pulled out the cornerstone on which the system of Ptolemy had been erected. Such an extreme change in an axiomatic system and resultant model is called a paradigm shift. At the same time, we see that he retained the second axiom and continued to use circles to describe the paths of the planets. As we shall see, each planet actually travels in an elliptical path.

One problem with the Copernican system was that the concept of a moving Earth reintroduced the mystery of a lack of visible stellar parallax. The stars appeared to be permanently attached to some rigid unchanging latticework—an impossibility for close stars unless Earth was forever immobile. Copernicus claimed, correctly as it turned out, that the stars were too distant to exhibit parallax. Determining distances to celestial objects continues to be one of the prime challenges in astronomy. In fact, parallax caused by Earth's motion is so subtle that even 200 years after Galileo first turned a telescope skyward, the first reliable stellar distance was still a mystery. Finally, in 1838, **Friedrich Wilhelm Bessel** determined a stellar parallax angle of a star. From this angle, we can then obtain a distance in the following manner.

Example 2

Because Earth does, in fact, move in orbit around the sun, two pictures can be taken of a nearby star 6 months apart, and an angle of a right triangle can sometimes be inferred from the pictures by measuring the parallax shift of the star in question among the background stars. (Recall the analogous situation, discussed in the last section, created by extending your arm and siting your thumb against a more distant object with each eye separately. A picture is being taken by your brain from two

stellar parallax One-half of the angular displacement in the apparent position of a star when observed from points on Earth's orbit that are separated by 180°.

vantage points, and each one displays your thumb differently among the background objects.) The actual angle that astronomers call **stellar parallax** is the angle p in **Figure 4.2.10**.

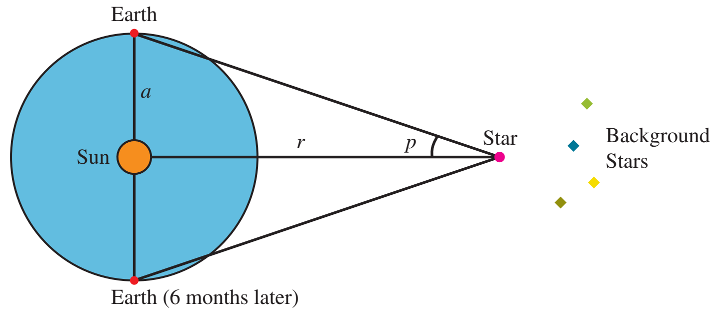


FIGURE 4.2.10 Finding the parallax angle.

The distance r to the star under consideration is quite large in comparison to the Earth–sun distance a, and therefore p is extremely small. For instance, the nearest star, Alpha Centauri, has a stellar parallax of only 0.76″, much less than the angle subtended by the thickness of a piece of paper held at arm's length. Convenience thus dictates that we measure p by using seconds of arc. Therefore, we can use the small-angle formula from the last section to write

$$\frac{p}{206,265} = \frac{a}{r}.$$

Because $a = 1$ AU, we can isolate r in the above equation to get distance as a function of parallax:

$$r = \frac{206,265}{p}.$$

parsec The distance to an object at which the stellar parallax has a measure of 1 arcsecond (1″). It is equal to 3.26 light-years (ly).

This computes distance in terms of astronomical units and yields large, unwieldly values for r. Instead, astronomers define 1 **parsec** as the distance at which a star possesses a parallax of 1 second of arc. This forces 1 parsec (abbreviated pc) to be equal to 206,265 AU. With these convenient units, we may determine the distance to a star in parsecs simply by inverting its stellar parallax, measured in arcseconds:

$$r = \frac{1}{p}.$$

The above-mentioned Alpha Centauri, for example, is located at a distance

$$r = \frac{1}{0.76} = 1.31 \text{ pc}.$$

Because 1 pc = 3.26 ly, this translates to 1.31(3.26) = 4.3 ly. The insertion of the Hubble Telescope into Earth's orbit increased the number of stars with measurable parallax and so extended the utility of this once rather limited method. ♦

We pause at this point to observe that one of the driving forces behind the lifelong motivation of Copernicus was the further glorification of God. He felt that the establishment of a simpler system for the universe was, in fact, bold evidence of a divine creation. It is interesting at this point to note that neither Ptolemy nor Copernicus made any definite claims in their defining works—the *Almagest* and *De Revolutionibus*—concerning the physical reality of the systems they set forth. Each of these books reveals a mathematical structure for simulating the planetary motions and making predictions, but does not address whether these structures, in truth, describe the nature and arrangement of celestial spheres. It is an example of a long-standing debate that continues today: Does every piece of mathematics have an existent representation somewhere in the universe, or is most of it simply the creation of the human mind? One person who was very much a realist—a believer that his mathematics provided an accurate description of a real situation—was born shortly after Copernicus died.

Few are the people who persevere in the face of constant tragedy to achieve significant accomplishments, but **Johannes Kepler** (1571–1630) is an example of Shakespearean proportions. (See **Figure 4.2.11**.) Kepler was born into poverty in southwestern Germany. His father was a brutal man who disappeared entirely while Johannes was still quite young. Kepler's mother was no jewel either. By his own account, she was "swarthy, gossiping, and quarrelsome, of a bad disposition." Later in her life, she required a defense by her son in a 3-year trial on charges of witchcraft. She barely escaped being burned at the stake.

FIGURE 4.2.11 Johannes Kepler on a Polish stamp.

Kepler himself was a frail and sickly child whose life was fraught with health problems. His intellectual prowess at the local public school, however, led to young Johannes being sent to the seminary in 1584 and to the University of Tübingen 4 years later. It was during his time at this famous school that Kepler began to resonate with an energy and a curiosity to understand the universe. He became convinced that the world and its attendant mysteries were knowable in a form intricately woven in mathematics by the hand of God. He wrote:

Geometry existed before the Creation. It is co-eternal with the mind of God. . . . Geometry provided God with a model for the Creation. . . . Geometry is God Himself.

Kepler's laws of planetary motion

1. The orbit of each planet is an ellipse, with the sun at one focus.
2. The line joining a planet and the sun sweeps out equal areas in equal amounts of time.
3. The square of the period of a planet's revolution is proportional to the cube of its mean distance from the sun.

At Tübingen, the astronomer Michael Maestlin privately introduced his eager star pupil to the principles of Copernicism. Although Kepler was reluctant to leave his theological studies, his immersion into mathematics and astronomy qualified him for a mathematics teaching position at Graz in Austria in 1594, which he accepted out of financial need. Six years later, he was hired as the assistant to the Imperial Mathematician at the court of Emperor Rudolf II in Prague. Kepler's first assignment was to analyze a large set of observational data of the planets and to determine, once and for all, the elusive orbit of Mars. It was during his 4-year effort to accomplish this task that he made two of the three great discoveries later to be known as **Kepler's laws of planetary motion**.

The path of nearby Mars had always invoked speculation, partly because it displayed a relatively large deviation from circular motion. From the data at Prague, Kepler determined a shape for Mars's orbit that indicated it to be noncircular. Additionally, as both a mystic and a realist, Kepler had long felt that the sun played a key role in causing the movements of all the planets through some type of attraction. This firm belief, along with the approximate curve he had produced, prodded him into the eventual realization that Mars orbited the sun not along a circle but along another of the conic sections—an *ellipse*. This startling conclusion is now known as **Kepler's first law:**

The orbit of each planet is an ellipse with the sun at one focus.

Led by his intuition, Kepler had replaced one noble curve of geometry with another of equal standing. He had discovered a true law of nature and, in the process, had accomplished a most cherished goal. The significance of this event is hard to overestimate. Owen Gingerich, the renowned historian of astronomy at the Harvard-Smithsonian Institute for Astrophysics, said in *Circle of the Gods: Copernicus, Kepler, and the Ellipse* that

> As Newton later claimed, Kepler had guessed—but it surely was an inspired guess based on the latest observations of previously unavailable quality and quantity. The circles of the gods had finally failed.

ellipse The set of all points whose sum of distances from two fixed foci is constant.

focus One of the two fixed points inside an ellipse, from which the distances to any point on the ellipse total a constant value.

Ellipses and their beautiful properties had been known since the days of Hipparchus. Any school child can draw one by simply taking up the slack with a pencil in a piece of string whose endpoints F_1 and F_2 have been fixed. One then keeps the string tight while tracing out a closed curve with the pencil. The resulting curve is called an **ellipse**, and each of the points F_1 and F_2 is called a **focus**. (See **Figure 4.2.12**.) Note that this implies that we can define an ellipse as the set of all points whose sum of distances from two fixed foci is constant (i.e., the length of the string).

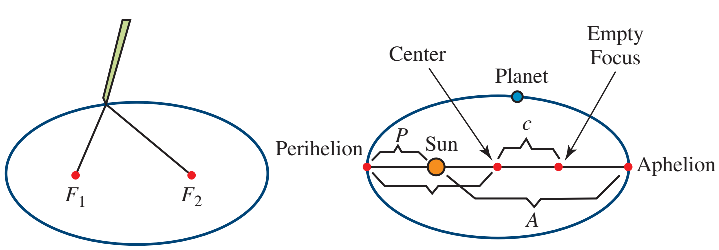


FIGURE 4.2.12 Kepler's first law.

Example 3

eccentricity Numerical value between 0 and 1 indicating the extent to which an ellipse departs from a circle.

perihelion The point in the orbit of an object in our solar system where the object is at a minimum distance from the sun.

aphelion The point in the orbit of an object in our solar system where the object is at a maximum distance from the sun.

The extent to which an ellipse deviates from being a true circle is measured by a parameter known as its **eccentricity**. (I'm sure you all know someone who deviates enough from normal to be labeled an eccentric.) The *center* of an ellipse is the midpoint of the line that goes through the two foci, known as the *major axis*. If c represents the distance from the center to either focus and a represents one-half the length of the major axis, then the eccentricity e is defined by the ratio $e = \frac{c}{a}$. Note immediately that it must be true that $0 \le e < 1$ and also that the closer the foci are to each other, the smaller the values of c and e. An ellipse having $e = 0$ is a circle. On the other hand, values for e close to 1 are indicative of stretched out, cigar-shaped ellipses. Because all the planets travel in near-circular paths, their orbits have relatively low eccentricity, and this contributed to the cosmic mystery for some time. (On the other hand, many comets, such as Halley's comet, have quite elongated elliptical orbits around the sun, with eccentricities typically greater than 0.9.)

It turns out that eccentricity is more easily determined by using the distances P and A to the points in a planet's orbit that are closest to the sun (labeled the **perihelion** by Kepler) and farthest from the sun (**aphelion**). Considering Figure 4.2.12, we see that

$$e = \frac{c}{a} = \frac{2c}{2a} = \frac{A - P}{A + P}.$$

The often-quoted figure of 150 million km for the distance from Earth to the sun is really the value of the parameter a, referred to as the *mean distance*. In reality, Earth achieves aphelion 152 million km from the sun and perihelion at 147 million km, yielding an eccentricity for our home planet's orbit of

$$e = \frac{152 - 147}{152 + 147} = \frac{5}{299} = 0.0167. \quad ♦$$

Kepler was not finished with his contributions. In 1609, he published his results in a book called *Astronomia Nova* (*The New Astronomy*), which gives highly mathematical demonstrations for his claim of elliptical orbits. One famous result is now referred to as **Kepler's second law.**

The line joining a planet and the sun sweeps out equal areas in equal amounts of time.

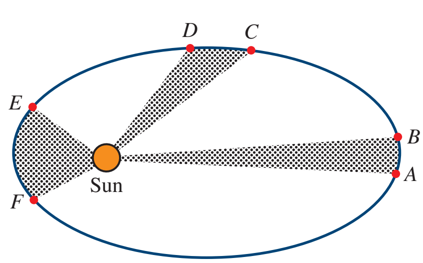


FIGURE 4.2.13 Kepler's second law.

Most astronomers up to this point in history had always felt that the speed of each planet was uniform. (Ptolemy was a notable exception. He used varying speeds for his deferents.) Note how Kepler's second law implies that the speed of any planet is continuously changing. In **Figure 4.2.13**, each of the shaded regions has the same area, and each of the arclengths AB, CD, and EF are traveled by the planet in the same amount of time. The planet travels slowest along the path AB because that has the shortest of the three lengths; it goes faster along CD and faster still along EF. I think you can imagine the body accelerating from aphelion to perihelion, achieving a maximum speed there, and then slowing down after it rounds the curve and returns to aphelion, where it achieves its minimum speed before starting the whole trip over again.

Kepler was not quite done. In his final hurrah, he published *Harmonice Mundi* (*The Harmony of the World*) in 1619, a grand monument to the labors of his life in which he attempted to unify astronomy, astrology, music, and geometry in the ultimate explanation of the universe. Although most of it was fantastical musings, one final major planetary rule, **Kepler's third law**, was revealed.

> **The square of the period of a planet's revolution is proportional to the cube of its mean distance from the sun.**

In equation form, this is written.

$$p^2 = ka^3,$$

where p is the period and a represents the same value as in our definition of eccentricity—one-half the length of the major axis of the planet's orbit. If we choose years for our time measurement and astronomical units for distance, our constant of proportionality has a convenient value of 1. So we get simply

$$p^2 = a^3.$$

Example 4

Consulting Figure 4.2.8, we find that the mean distance a of Mercury from the sun is 0.387 AU. Thus,

$$p^2 = (0.387)^3 = 0.0580 \Rightarrow p = \sqrt{0.0580} \approx 0.24 \text{ yr.}$$

This value matches up with that in the table. Also, because it is true that $p = a^{3/2}$ and $a = p^{3/2}$, note that we can now write the Copernican velocity v as a function of a.

$$v = \frac{2\pi a}{p} = \frac{2\pi a}{a^{\frac{3}{2}}} = \frac{2\pi}{\sqrt{a}} \text{ AU/yr.}$$

Recall that this formula is based on the assumptions of the planet moving at a constant speed in a circular motion. Because Kepler revealed both of these assumptions to be false, the Copernican velocity now becomes simply a good estimate of the *average velocity* of the planet. If we wish to convert the units to kilometers per second, recall that we need to multiply by the conversion factor of 4.76.

$$v = \frac{2\pi}{\sqrt{a}}(4.76) = \frac{9.52\pi}{\sqrt{a}} \text{ km/s.} \quad \blacklozenge$$

So the penetrating insight and imagination of Johannes Kepler had arrived at three of the most famous results in the search to understand our universe. In so doing, he provided substantial momentum to the revolution begun by Copernicus. Kepler himself, never one to shy away from the dramatic statement, proclaimed in *The Harmonies of the World* that

> With this symphony of voices man can play through the eternity of time in less than an hour, and can taste in small measure the delight of God, the Supreme Artist. . . . I yield freely to the sacred frenzy. . . the die is cast, and I am writing the book—to be read either now or by posterity, it matters not.

Name ____________________

Exercise Set **4.2**

Note: Answers should be given using the appropriate number of significant digits (cf. Appendix).

1. What student of Plato used lunar eclipses to demonstrate that Earth was a sphere?

2. The epicycles and deferents used by Ptolemy in his model accounted for what type of unusual motion displayed by the planets?

3. Approximately how long was the Ptolemaic system used as a model for our planetary system?

4. What major paradigm shift was achieved by Nicholas Copernicus in his book *De Revolutionibus?*

5. Kepler correctly postulated in his first law that each planet travels around the sun according to what type of orbital curve? What position does the sun occupy relative to this curve?

6. What is Kepler's second law?

Use the following table for computing the Copernican velocities (in kilometers per second) of the following four planets.

Planet	Mean Distance (AU)	Period (yr)
Venus	0.723	0.62
Earth	1.00	1.00
Jupiter	5.20	11.87
Saturn	9.54	29.46

7. Earth

8. Venus

9. Jupiter

10. Saturn

11. Altair is the brightest star in the constellation Aquila. If it has a parallax of 0.20″, what is its distance in parsecs? In light-years? In astronomical units?

12. The brightest star in the sky is Sirius, and it is easily located in the early evenings of January and February. If it has a parallax of 0.38″, what is its distance in parsecs? In light-years? In astronomical units?

13. If a star is located 23 ly from Earth, what is its parallax angle?

14. If a star is located 48.5 ly from Earth, what is its parallax angle?

15. How many kilometers equal the distance of 1 pc?

16. The Andromeda galaxy is about 2 million ly from Earth. Do you think this distance estimate was found by the parallax method? Explain.

17. Take a piece of string 8 inches (in.) long, and use it to draw three ellipses. First, fix the endpoints of the string 6 in. apart, and call those points *A* and *B*. Take up the slack in the string with your pencil and, keeping the string tight, let it guide your hand as you sketch an ellipse. Repeat this procedure with *A* and *B* only 4 in. apart and then again with a 1-in. spread. Which ellipse most closely resembles a circle? Which one has the smallest eccentricity? The greatest? Which would be most likely to represent the path of a planet orbiting the sun?

Use the following table for computing the eccentricities of the following four planets.

Planet	Perihelion (AU)	Aphelion (AU)
Mercury	0.3060	0.4670
Venus	0.7184	0.7282
Mars	1.381	1.666
Jupiter	4.951	5.455

18. Mercury

19. Venus

20. Mars

21. Jupiter

22. Neptune has an orbital eccentricity of 0.0100. If its aphelion distance from the sun is 30.4 AU, determine its perihelion distance.

23. One of the reasons Pluto lost its classification as a planet is that its orbital eccentricity of 0.2484 is larger than that of all the remaining eight planets. (Pluto is now called a dwarf planet.) If its aphelion distance from the sun is 49.2 AU, determine its perihelion distance. Compare these values to those of Neptune in the previous exercise. What conclusion do you reach?

24. Many comets orbit the sun in a highly eccentric elliptical path, revealing themselves only periodically to observation from Earth. Comet Halley has perihelion and aphelion distances of 0.587 and 35.3 AU, respectively. Find the eccentricity of its orbit.

25. Comet Encke has a perihelion distance of 0.332 AU and an eccentricity of 0.8499. Find its aphelion distance.

26. Explain why we may conclude from Kepler's second law that the speed of each planet varies, reaching a maximum at perihelion and a minimum at aphelion.

27. Assuming that the mass of a moon revolving around a planet is significantly less than that of the planet, the moon's orbital path is an ellipse with the center of the planet at one focus. In this case, the point of greatest distance is known as the *apogee*, and the point of least distance is called the *perigee*, usually measured from the center of the planet to the center of its moon. The eccentricity of the orbit is again given by $e = \frac{A-P}{A+P}$, where *A* and *P* are the apogee and perigee distances, respectively. In Example 5 from the first section, the distances to perigee and apogee of our moon were 363,000 and 405,000 km, respectively. What is the eccentricity of our moon's orbit?

28. Kepler's laws of planetary motion apply to any body in our solar system—all the asteroids, comets, and planets discovered since Kepler's time. Use the third law to fill in the blanks below. Do the values for Jupiter and Saturn match up with Figure 4.2.8?

Planet	Mean Distance (AU)	Period (yr)
Jupiter	5.20	
Saturn		29.5
Uranus	19.19	
Neptune		164.8
Pluto	39.44	
Comet Halley		76.0
Comet Encke	2.21	

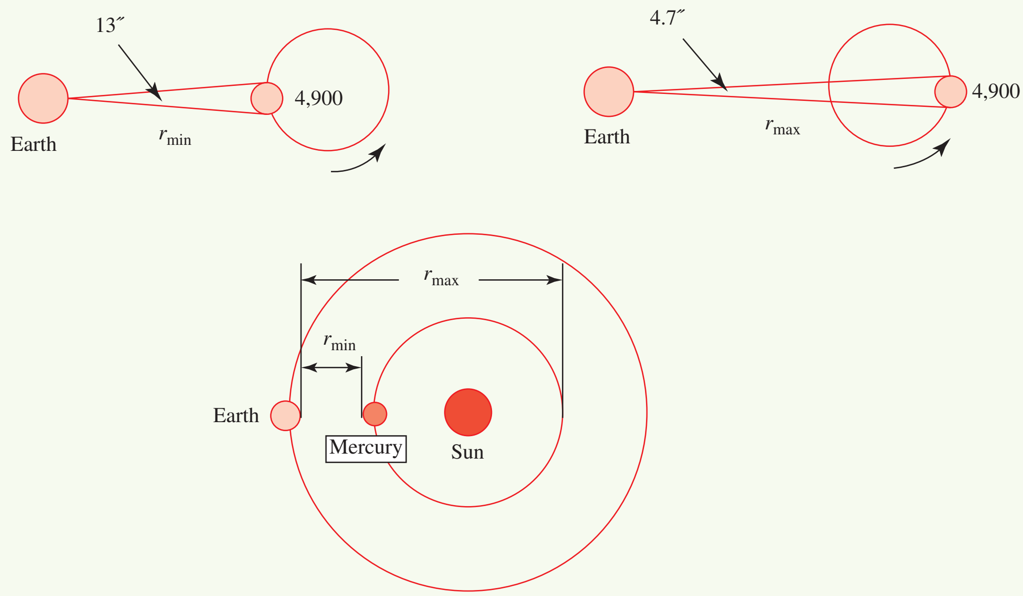
29. Express the Copernican velocity (in kilometers per second) as a function of the period *p* (yr) of a planet. Use this function and the periods computed in the previous problem to find the velocities for Uranus, Neptune, and the dwarf planet Pluto.

30. In Figure 4.2.12, we can see that the sum of the distances to aphelion and perihelion must be equal to twice the mean distance. Symbolically, $2a = A + P$. If the mean distance to Uranus is 19.2 AU and the perihelion distance is 18.3 AU, find the aphelion distance and the eccentricity of its orbit.

31. The comet Hyakutake was sighted for the first time in January 1996 and was bright enough in the evening sky by March to be seen with the naked eye. In fact, this brightness caused it to receive a great deal of media attention even though about a dozen comets are sighted each year. Its period was determined to be about 9,100 yr. Use Kepler's third law to find its mean solar distance.

32. Perihelion distance P to Comet Hyakutake was observed to be about 0.20 AU. Use the mean distance a computed in the previous exercise to find the aphelion distance. (*Hint*: From Exercise 30, we know that $2a = A + P$.)

33. Although each planet varies in brightness over time as a result of its changing distance from Earth, this is a phenomenon that cannot be perceived by the naked eye, and so pretelescopic astronomers were unaware of it. Modern telescopes reveal that the angular diameters of Mercury range from a minimum of 4.7″ to a maximum of 13″. Given that Mercury has a linear diameter of 4,900 km and approximating its orbit by a circle, we can estimate the (mean) diameter of the orbit. Find the minimum distance r_{min} (corresponding to 13″) and the maximum distance r_{max} (corresponding to 4.7″) from Earth to Mercury as in the accompanying figure, and subtract them to get the orbital diameter. (Recall that the angular the diameter α and distance r are related by $\frac{\alpha}{206{,}265} = \frac{4{,}900}{r}$ when α is measured in arcseconds.)



34. The angular diameter of Mars ranges from a minimum of 4″ to a maximum of 24″. If Mars has a linear diameter of 6,800 km, estimate the mean diameter of its orbit.

35. Venus presents a disk of 10″ at full phase up to 60″ at crescent phase. If it has a linear diameter of 12,000 km, estimate the mean orbital diameter of Venus.

36. The minimum and maximum angular diameters of Jupiter are about 30″ and 50″, respectively. If Jupiter has a linear diameter of 144,000 km, estimate the mean orbital diameter.

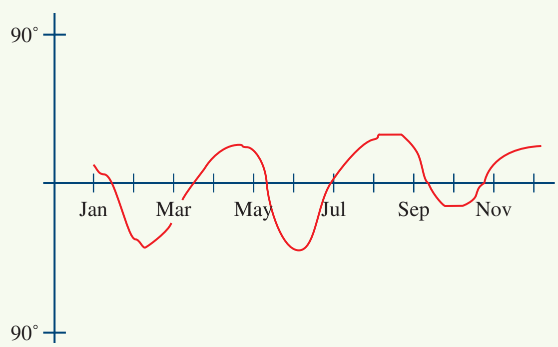
37. The Titius–Bode rule is an algorithm for producing a sequence of numbers that eerily coincide with the average distances from the sun of the first seven planets (but not for Neptune). The sequence is generated by first writing 0, 3, 6, 12, 24, and so on, doubling each successive number. Next, 4 is added to each number, and then the result is divided by 10. This gives a predicted distance in astronomical units that matches up with the actual distance with remarkable accuracy. The dwarf planet Pluto and the asteroids are also included in the following list.

Planet	**Titius–Bode Prediction** (AU)	**Actual** (AU)
Mercury	(0 + 4)/10 = 0.4	0.387
Venus	(3 + 4)/10 = 0.7	0.723
Earth	(6 + 4)/10 = 1.0	1
Mars	(12 + 4)/10 = 1.6	1.524
Asteroid belt	(24 + 4)/10 = 2.8	2.77 average
Jupiter	(48 + 4)/10 = 5.2	5.203
Saturn	(96 + 4)/10 = 10.0	9.539
Uranus	(192 + 4)/10 = 19.6	19.18
Neptune		30.06
Pluto	(384 + 4)/10 = 38.8	39.44

Nobody really knows whether these two lists match so closely by coincidence or if there is really a physical reason for the connection. Evidence against would be that Neptune does not follow suit. However, it is worth noting that the rule was first devised by Johann Titius in 1766 before the discovery of either the asteroid belt or Uranus. (Jupiter's tremendous gravity probably kept the thousand or so asteroids occupying the orbit between Mars and Jupiter from coalescing into a planet.) If you were to suggest a solar distance to look for another planet, what would it be? If you use Kepler's third law, what would be the period of this planet?

38. For satellites in orbit close to Earth's surface, information is often given in terms of heights above the surface. One can find eccentricity by using $e = \frac{A-P}{A+P}$ by remembering to add Earth's radius to determine the apogee and perigee. Sputnik I, orbited by the Russians in October 1957, had 583 and 132 mi above Earth's surface as the highest and lowest points of its elliptical orbit. What is the eccentricity of this orbit? (Use 4,000 mi as the radius of Earth.)

39. This graph approximately represents the elongation angles of Mercury as a function of the day of the year. Note that the values of the local maxima and minima differ. This is so because the orbit of Mercury has an eccentricity of 0.2056, the largest of the eight planets. Mercury is 0.47 AU from the sun at aphelion and 0.31 AU at perihelion. If Mercury happened to be at aphelion coincidentally with maximum elongation as seen from Earth, what would be the elongation angle? What would be the angle at perihelion? (See Section 4.1, Exercise 28.)



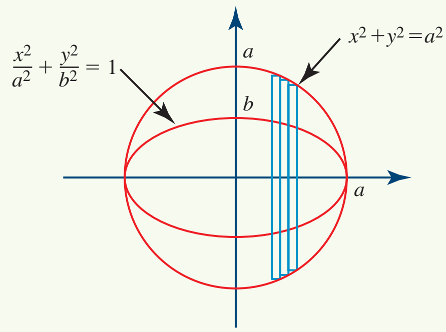
40. Kepler needed to know the formula for the area of an ellipse when studying the data that led to his second law. In his book *Calculus Gems*, George F. Simmons points out that a procedure similar to that of Kepler was used by his contemporary **Bonaventura Cavalieri** (1598–1647). When centered at the origin of a Cartesian coordinate system, an ellipse with semi-axes a and b and a circle of radius a have the respective equations

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad x^2 + y^2 = 1.$$

Solving both of these equations for y, we obtain

$$y = \pm \frac{b}{a}\sqrt{a^2 - x^2} \quad \text{and} \quad y = \pm\sqrt{a^2 - x^2}.$$

So we see that the y-coordinate on any point of the ellipse is b/a times the corresponding y-coordinate of the point on the circle having the same x-coordinate.



If we think of the areas as being approximated by a set of thin rectangles, then each rectangle used to compute the area of the ellipse has length b/a times the corresponding rectangle used to compute the area of the circle. Hence

$$\text{Area of ellipse} = \frac{b}{a}(\text{area of circle}) = \frac{b}{a}(\pi a^2) = \pi ab.$$

What is the area of the ellipse with semi-axes of lengths 8 and 12? What is the area of an ellipse with semi-axes of lengths 15 and 28?

41. If the orbit of a planet has eccentricity e and mean distance (major semi-axis length) a, then the length of the minor semi-axis is $b = a\sqrt{1-e^2}$. Use the formula in the previous excercise to find an expression for the area of the ellipse in terms of a and e. For orbits with low eccentricity, does this differ much from the area of a circle of radius a?

4.3 The Defense of Copernicanism

In medicine it was sufficient to quote Galen, as to quote Aristotle in practically everything else. For Galileo, to quote was not sufficient; he turned to mathematics.

—George Polya

On February 17, 1600, after eight years of imprisonment, the Dominican monk **Giordano Bruno** (b. 1548) was burned at the stake in Rome for heresy, a victim of the Roman Inquisition. Although he was charged with several other religious offenses, his advocacy of Copernicanism has often been used to proclaim Bruno the first martyr of the new astronomy. His execution and the subsequent persecution of Galileo Galilei have been commonly cited as prime examples of the stubborn resistance of organized religion to the noble advance of science. A closer examination, however, reveals a more balanced perspective.

Bruno was not a scientist; he endorsed the heliocentric theory only as one of many arguments to be used in his efforts to reconcile Protestants and Catholics in a time of violent religious warfare. He had a poor technical understanding of *De Revolutionibus*, yet he seized its ideas as a grand philosophical metaphor. The important thing to Bruno was that Earth had been shown not to be at the center of the universe. This centerless universe not only must be of infinite extent, but also must contain an infinite number of other worlds because Earth now had no claim to uniqueness.

Unfortunately, the condemnation of Bruno set the stage for the persecution of **Galileo Galilei** (1564–1642), one of the first true scientists and the valiant champion of Copernicanism. (See **Figure 4.3.1**.) His goals in life were completely different. Galileo vigorously supported the heliocentric theory based solely on years of scientific investigation and attached to it no religious implications. Bruno's shadowy rhetorical arguments lacked what Galileo accrued in abundance—conclusions based on solid experimental evidence. Historians of science credit Galileo and his contemporary, **Réné Déscartes**, with the reformulation of the composition of scientific activity. By molding a new methodology of experimentation and analysis, they forged a permanent bond between science and mathematics, one begun by Copernicus and Kepler. Galileo, in particular, insisted that

Courtesy of Jacob Bourjaily.

FIGURE 4.3.1 Galileo on an Italian banknote.

mathematical relationships, derived from experimentation and embodied by equations and functions, were the cornerstone to any theory worth postulating. His work paved the way for the astonishing successes of modern science in the last three centuries.

Prior to the seventeenth century, the approach to understanding nature had never varied much from that taken by the ancient Aristotle and his disciples—one in which explanations for natural phenomena were qualitative rather than quantitative. Actual experimentation either to initiate exploration of a new process or to corroborate the predictions of an existing theory was never seriously done. Aristotle claimed, for example, that a body twice as heavy as a second body must fall twice as fast, a conjecture no one ever bothered to check prior to Galileo.

By 1609, Galileo had been teaching mathematics at the University of Padua for 18 years. During this time, his research of the properties of pendulums and his important discoveries in mechanics—the physics of motion—had already earned him a glowing reputation. From his experiments, he formulated two laws giving the velocity and distance of a dropped object as functions of time alone, *regardless of the size or mass of the object*.

Strictly speaking, the **velocity** of a moving object is specified by both the speed at which it is moving *and* the direction of movement. Going 30 miles per hour (mi/h) east on Main Street is a different velocity than going 30 mi/h west on Main Street. **Acceleration** of a moving object measures how the velocity changes per unit of time. This can be a change in speed, a change in direction, or both. If a moving object is subject to no acceleration, then the velocity is a constant value V, and the distance traveled in time T is just the product VT. Let $v = V$ be the horizontal line in the coordinate system on the left in **Figure 4.3.2**. Then the distance traveled in time T is equal to the area of the rectangle under that line and over the interval $[0, T]$.

velocity The rate of change of position of a moving body, i.e., its speed and its direction of motion.

acceleration Rate of change of velocity with respect to time. It has both a numerical value and a direction.

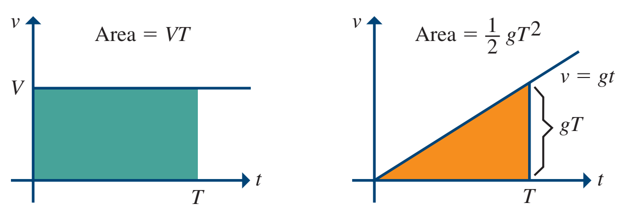


FIGURE 4.3.2 Distances traveled by a moving object can be represented as areas. The area on the left corresponds to an object moving at constant speed $v = V$. The area on the right corresponds to an object moving at speed $v = gt$.

Galileo observed that falling bodies gain speed as they fall. He speculated that the acceleration of a body dropped from rest was constant in both numerical value and direction (toward the center of Earth). He then experimented to verify this relationship by rolling balls down inclined planes—a related phenomenon of motion that facilitated the necessary measurements. Today, we represent this constant by the letter g for acceleration due to gravity near the surface of Earth (or whatever planet where you may be). If v is the velocity of a dropped object after a time t from the moment of release, then

$$v = gt.$$

The graph of this linear function is shown on the coordinate system on the right in Figure 4.3.2. Because the velocity is continuously changing, we cannot just multiply a velocity by a time to obtain a distance. But notice that after a specific elapsed time T, the velocity is $v = gT$. By again equating the distance d traveled as the area under the curve as before, we compute d as the area $\frac{1}{2}gT^2$ of the right triangle with legs T and gT. Writing d as function of any arbitrary time t, we have

$$d = \frac{1}{2}gt^2.$$

These two functions give the velocity and distance of a dropped object as a function of time after release near the surface of any planet (neglecting the effects of air resistance). Of course, the constant g (sometimes referred to simply as *surface gravity*) depends on the planet. A complete list of values is given in **Figure 4.3.3**.

Planet	**Acceleration g (m/s²)**
Mercury	3.8
Venus	8.9
Earth	9.8
Mars	3.7
Jupiter	24.9
Saturn	10.4
Uranus	8.8
Neptune	12.0
Pluto	0.7

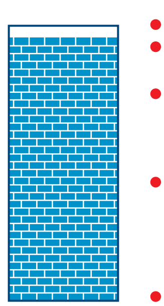
FIGURE 4.3.3 Acceleration due to gravity near the surfaces of the eight planets plus Pluto.

Example 1

The value of g for Earth has been determined to be about 9.8 meters per second (m/s) every second. (These units are abbreviated as m/s^2.) If a baseball takes 4 s to drop from the top of a building, how tall is the building, and how fast is the ball traveling when it hits the ground? (This is referred to as the *impact velocity*.)

Solution

The ball will have an impact velocity directed straight down of $v = 9.8(4) = 39.2$ m/s. The height of the building must be equal to the distance traveled by the ball: $d = ½(9.8)(4)^2 = 78.4$ m. Note the intermediate values for v and d in **Figure 4.3.4**.



t (s)	w (m/s)	d (m)
0	0	0
1	9.8	4.9
2	19.6	19.6
3	29.4	44.1
4	39.2	78.4

FIGURE 4.3.4 Velocity and distance of a falling object. ♦

mass Property of an object that is a measure of the amount of matter in the object. It can be thought of as a measure of its inertia.

grams The basic measure of mass in the metric system.

kilogram 1,000 grams.

weight Amount of force that a body exerts on the surface of a planet or moon as a result of gravity.

newton Unit of force in the metric system.

It is important to note that neither of Galileo's functions for velocity or distance incorporate the *mass* of the falling body. The **mass** of an object is the measure of the amount of matter of which it is composed. This value never varies with respect to the location of the object—it is the same on Earth, on the moon, or in the vacuum of space. Mass is measured in **grams** (g) in the metric system. One **kilogram** (1 kg) is 1,000 g.

We must be careful to never confuse mass with the separate concept of *weight*. An object's **weight** is the force with which it is attracted to the body on which it is resting. You weigh less on the moon, for example, because it attracts you with less force than Earth does. Force (to be discussed further in the next section) is measured using pounds in the English system or newtons in the metric system. One pound is the equivalent of about 4.45 newtons (N). One **newton** is defined as the amount of force required to give an acceleration of 1 m/s^2 to a mass of 1 kg. Because gravity on the surface of Earth is 9.8 m/s^2, it gives a weight (force) of 9.8 N to any object having a mass of 1 kg, a weight of 19.6 N to a mass of 2 kg, and so on. In general, we can write the relationship between the mass m and weight W of any object on a planet with surface gravity constant g as

$$W = mg.$$

Example 2

If you weigh 160 lb on Earth, what is your mass? What would your weight be on Mars?

Solution

Because mass is almost always expressed with metric units, we first determine your metric weight to be (160 lb)(4.45 N/lb) = 712 N. We substitute this for W in $W = m(9.8)$ to obtain

$$712 = m(9.8)$$

$$m = \frac{712}{9.8} = 72.7\text{kg}.$$

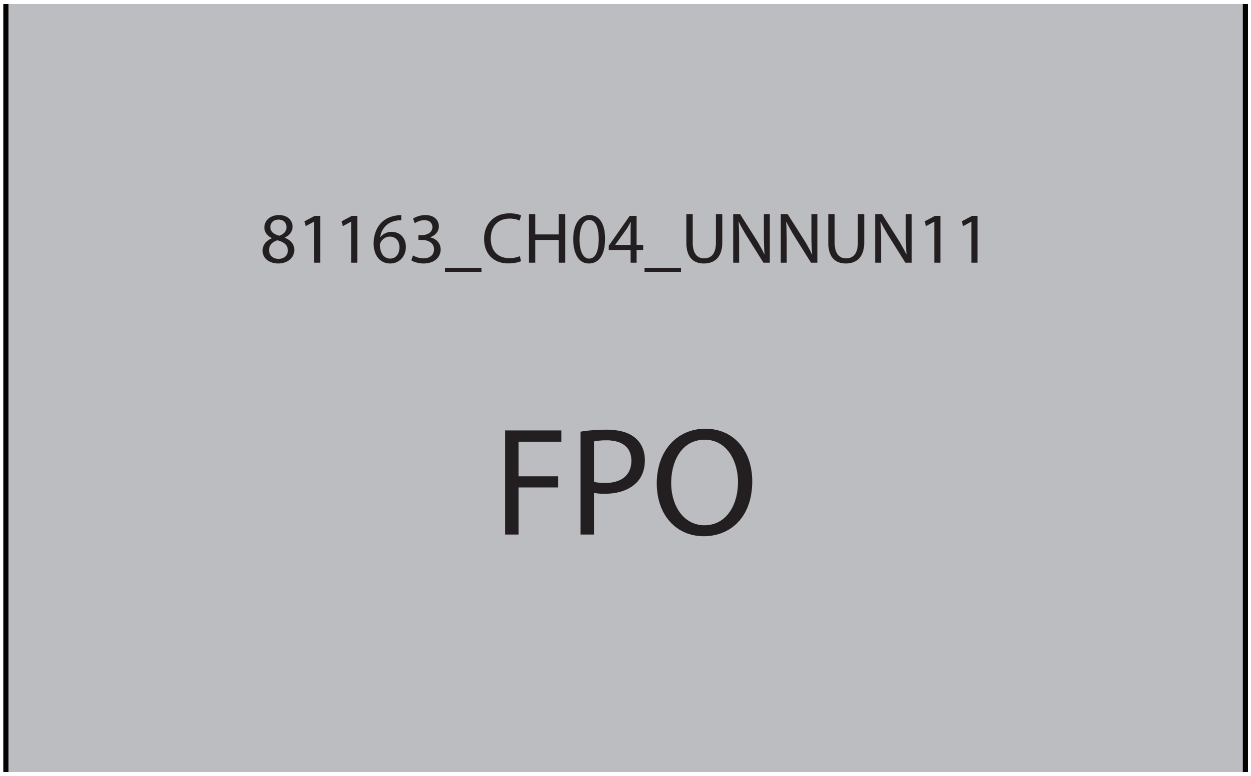
(Note that this says that a 160-lb object has a mass of 72.7 kg. Pounds are often mistakenly used as units of mass with a so-called conversion factor of 2.2 lb/kg. See Exercise 10.)

Your mass, of course, remains the same everywhere (assuming you've laid off the pasta), but the acceleration due to gravity on the smaller Mars is 3.7 m/s^2. Thus, your weight there would be $W = mg = 72.7(3.7) = 269.0$ N. Converting back to English units, we divide 269.0 N by 4.45 N/lb to get the equivalent weight of 60.4 lb. ♦

Because the surface gravity on Mars exerts just $\frac{3.7}{9.8}$ of the pull that it exerts on Earth, we also could have found your Martian weight in the above example by simply multiplying: $\left(\frac{3.7}{9.8}\right)160 = 60.4$ lb. In general, if your weight is W_E on Earth and W_P on planet P with surface gravitational acceleration g, then

$$\frac{W_P}{W_E} = \frac{mg}{m(9.8)}$$

$$W_P = \left(\frac{g}{9.8}\right)W_E.$$



Courtesy of Universal Uclick

Example 3

Garfield weighs 28 lb. What would he weigh on Venus? On Jupiter?

Solution

Consulting Figure 4.3.3, we see that Garfield would weigh

$$W_{\text{Venus}} = \left(\frac{8.9}{9.8}\right)(28) = 25 \text{ lb on Venus}$$

and

$$W_{\text{Jupiter}} = \left(\frac{24.9}{9.8}\right)(28) = 71 \text{ lb on Jupiter.} \quad \blacklozenge$$

As significant as his achievements were in the field of physics, Galileo acquired his greatest fame (and got into the most trouble) for his defense of the Copernican system. At the stately age of 45, his rather orderly life as a university professor took a dramatic turn when he learned of a wondrous new invention, the telescope. Immediately, he constructed one of his own and began a systematic study of the night skies, which was the first use of this scientific instrument in astronomy. The impact was dramatic. *Everything* in the heavens upon which Galileo turned his scope was being observed for the first time. In 1610, Galileo published a book of his studies, *Sidereus Nuncius*, which rocked the scientific and religious worlds. As a title, he wrote

> ASTRONOMICAL MESSAGE
> Containing and Explaining Observations Recently Made,
> With the Benefit of a New Spyglass, About the
> Face of the Moon, the Milky Way, and Nebulous
> Stars, about Innumerable Fixed Stars and also Four
> Planets hitherto never seen, and named
> MEDICEAN STARS

Galileo continued to give more detailed descriptions of these findings, two of which are of particular interest to us here. Early in the book he commented on the craggy surface of the moon:

> . . . we certainly see the surface of the Moon to be not smooth, even, and perfectly spherical, as the great crowd of philosophers have believed about this and other heavenly bodies, but, on the contrary, to be uneven, rough, and crowded with depressions and bulges. And it is like the face of the Earth itself. . . .

This description flew in the face of the standard Aristotelean view that all the heavenly bodies must, by nature and God, be perfectly round and smooth. To Galileo, the moon offered convincing evidence that Earth was not unique but rather just one of the collection of rocky orbs that circled the sun. His newly discovered objects supported that notion as well. On January 7, 1610, he had noticed three peculiar stars along a straight line through Jupiter, two near to the planet on the east and one on the west. When he looked again the next night, he noticed the same three stars but in a *different* arrangement. Intrigued, he began making nightly sketches of this mysterious behavior and soon realized from their varying patterns that these were not stars at all, but four new bodies (he later sighted a fourth) in orbit *around Jupiter*. This conclusion was a stunning discovery, for it proved the existence of a second center of motion, thereby providing a disclaimer to Earth's special status in that role. This belief was falsified by the finding of a planet that clearly retained its satellites as it traveled through space.

Galileo had privately long believed in the heliocentric theory, but now, as the evidence piled up, he became a fervent torchbearer for its validity. Any lingering doubts were forever banished when he turned his telescope on Venus and observed that the planet went through a full series of phases just as our own moon does each month. This is possible in a Copernican scheme, because Venus would appear more than half-illuminated (called a

gibbous A phase of an illuminated body in which more than one-half of the illuminated side is visible from Earth.

crescent A phase of an illuminated body in which less than one-half of the illuminated side is visible from Earth.

gibbous phase) when it was farther from us than the sun. It is impossible if Venus is riding on an epicycle between the sun and Earth, because most of the side illuminated by the sun would always face away from Earth, revealing *only* a **crescent** phase. (See **Figure 4.3.5**.) It dealt the Ptolemaic system another shattering blow.

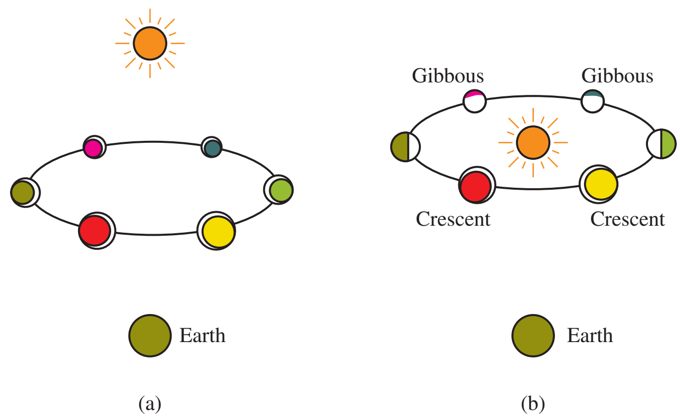


FIGURE 4.3.5 The phases of Venus as they would appear in (a) the Ptolemaic system and (b) the Copernican system.

Example 4

We must stop for a moment and consider the logic involved with using these observations in support of a particular planetary system. Galileo offered them as "proof" of the Copernican model. But is this a valid conclusion? An analogous situation would be the following syllogism.

Fact: If a car is a Porsche, then it is fast.
Hypothesis: Sabrina owns a Porsche.
Conclusion: Therefore, Sabrina owns a fast car.

In other words, being fast is a necessary consequence of being a Porsche. Is the reverse true, however? Is being fast sufficient to ensure being a Porsche?

Hypothesis: Rusty owns a fast car.
Conclusion(?): Therefore, Rusty owns a Porsche.

converse For the conditional statement "if *P*, then *Q*," the converse is "if *Q*, then *P*." Its truth value is not related to that of the original statement.

Any student of logic (and cars) will tell you that this is a false conclusion. A logician would say that the **converse** to a given true statement need not also be true. The converse to a statement such as "if *P*, then *Q*" is "if *Q*, then *P*." The converse to our above fact is false:

Converse: If a car is fast, then it is a Porsche.

For our present purposes, we are concerned with the validity of a planetary model. So we examine this statement:

Fact: If the model is heliocentric, then Venus will show many phases.

Galileo has observed the phases of Venus. Can he justifiably conclude that the system is heliocentric? No, he cannot. The converse to this fact is not true. Just as other types

contrapositive For the conditional statement "if *P*, then *Q*," the contrapositive is "if not *Q*, then not *P*." Its truth value is the same as that of the original statement.

hypothetico-deductive method A method of inquiry in which a theory gains increasing acceptance as more evidence for its validity is observed. This is *not* the same as a deductive method, by which a definitive conclusion is reached.

of cars may also be fast, there were other postulated schemes for the planetary structure that also predicted Venusian phases.

On the other hand, the **contrapositive** of a given fact must always be true. The contrapositive of the statement "if *P*, then *Q*" is "if *not Q*, then *not P*." In the above example, this would take the following form.

Contrapositive: If a car is *not* fast, then it is *not* a Porsche.

Galileo was quite correct in using the phases of Venus to eliminate the Ptolemaic system as a candidate, for it is the contrapositive of this true statement:

Fact: If the system is Ptolemaic, then Venus cannot show many phases. ♦

Hermann Weyl, the famous twentieth-century mathematician and physicist, once said, "Logic is the hygiene the mathematician practices to keep his ideas healthy and strong." Galileo was much too good a mathematician to not realize that his observations did not allow him to definitively *conclude* that Earth moves around a centralized sun; yet he was convinced this was the case. He was using what is now referred to as the **hypothetico-deductive method**—one in which a given hypothesis gains steadily increasing acceptance as it continues to pass a sequence of tests. Although Galileo was convinced of the truth of Copernicanism, the science of today usually does not profess to discover such absolutes. The phrase *current model* is the typical claim, that is, a mathematical framework that can be used for predictive purposes. A model is pronounced good if it gives accurate predictions. (Recall that neither Ptolemy nor Copernicus made any claims in their main astronomical works that their models represented the *real* structure of the cosmos.) The irony of Galileo's celebrated troubles with the Catholic Church is that it made only one substantial request that he declare his beliefs to be just that—a hypothetical model, convenient for use by mathematicians but *not* a description of physical reality. In 1616, the powerful Cardinal Roberto Bellarmino evaluated one of Galileo's analytical treatises in a letter to a friend:

> For to say that assuming the earth moves and the sun stands still saves all the appearances better than eccentrics and epicycles is to speak well. This has no danger in it, and it suffices for mathematicians.
>
> But to wish to affirm that the sun is really fixed in the center of the heavens and that the earth is situated in the third sphere and revolves very swiftly around the sun is a very dangerous thing, not only by irritating all the theologians and scholastic philosophers, but also by injuring our holy faith and making the sacred Scripture false.

This passage is indicative of the sympathies accorded Galileo by many leading theologians of the day, including Cardinal Maffeo Barberini, with whom he had many friendly and enlightening cosmological discussions. However, as in Example 3, they had legitimate doubts stemming from the potential fallacies in Galileo's logic. His results were simply not conclusive enough to force a reinterpretation of a centuries-old belief system. The stubborn Galileo did not agree. Even though he was a devout Roman Catholic, he was always fond of saying, "The Bible teaches how to go to heaven, not how the heavens go."

Unlike his shy predecessor Copernicus, Galileo was an egotistical and, at times, abrasive individual who loved to be at the center of controversy. As evidenced by the fate of

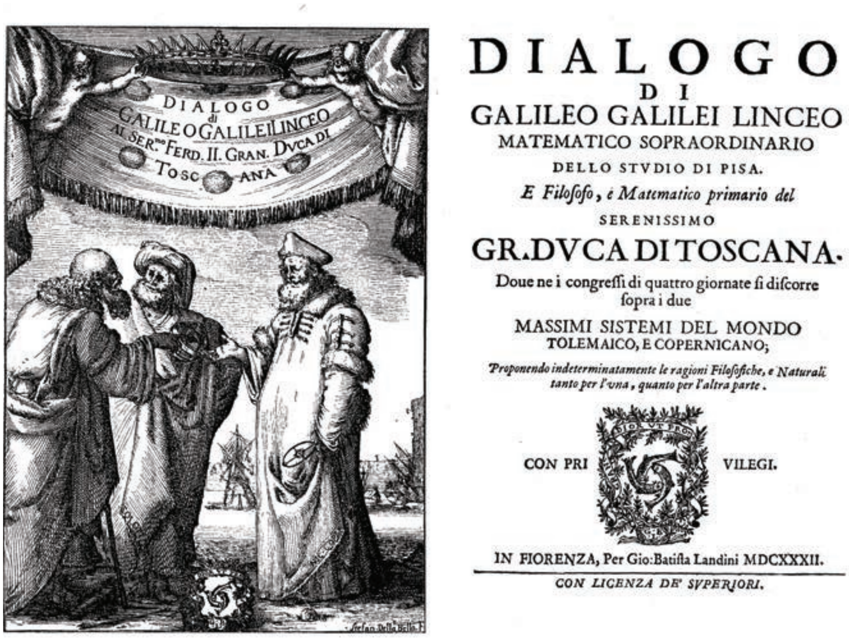


FIGURE 4.3.6 Frontispiece and first page of the *Dialogue*.

Bruno, however, the religious upheavals of the 1600s had created a tense atmosphere for the debate of the heliocentric theory in Italy. In 1616, *De Revolutionibus* was placed on the Index of Forbidden Books, and Galileo was formally warned by Cardinal Bellarmino against publicly defending the Copernican system. However, in 1623, his friend Cardinal Barberini was elected Pope Urban VIII. Filled with an increased sense of security, the headstrong Galileo overplayed his hand in 1632 when he published his most famous work, *Dialogue Concerning the Two Chief World Systems*. (See **Figure 4.3.6**.) Although it was billed by the author as an objective debate of the Copernican and Ptolemaic models, even the most casual reader realized the ***Dialogue*** to be an extensive argument in favor of Copericanism. Even worse, the enemies of Galileo seized upon the fact that one of the main characters, Simplicio, whom every reader knew also meant "simpleton," had essentially espoused the opinions of the pope himself. Such clear disrespect forced the hand of Pope Urban VIII, and in February 1633, Galileo was summoned to Rome at the age of 70 to face the Inquisition. Fourth among the stated charges listed against him was the following:

Dialogue The book written by Galileo, that defends the Copernican system through an extended conversation among three men.

The author claims to discuss a mathematical hypothesis, but he gives it physical reality, which mathematicians never do. Moreover, if defendant had not adhered firmly to the Copernican opinion and believed it physically true, he would not have fought for it with such asperity, nor . . . would he have held up to ridicule those who maintain the accepted opinion, and as if they were dumb mooncalves [*hebetes et pene stolidos*] described them as hardly deserving to be called human beings.

Indeed, if he had attacked some individual thinker for his inadequate arguments in favor of the stability of the Earth, we might still put a favorable construction on his text; but, as he holds all to be mental pygmies [*homunciones*] who are not Pythagorean or Copernican, it is clear enough what he has in mind. . . .

This leads us to believe that the trial was probably motivated primarily by the desire for public submission by the arrogant Galileo. Extreme punishment was not necessary, and Galileo probably understood from the beginning that a complete admission of his error was the inevitable conclusion. At the final point in the proceedings, the head of the inquisitors refrained from putting specific passages of the *Dialogue*, one at a time, to Galileo and demanding a refutation. This could have forced the issue of heresy back to the forefront, leading to a far more dangerous situation and a potentially crueler punishment. As it was, the next day his sentence read, in part, We pronounce . . . the said Galileo . . . have rendered yourself in the judgment of this Holy Office vehemently suspected of heresy, namely, of having believed and held the doctrine—which is false and contrary to the sacred and divine Scriptures—that the Sun is the center of the world and does not move from east to west and that the Earth moves and is not the center of the world. . . .

And, in order that this your grave and pernicious error and transgression may not altogether go unpunished . . . we ordain that the book of the "Dialogue of Galileo Galilei" be prohibited by public edict.

We condemn you to the formal prison of this Holy Office during our pleasure. . . .

Upon hearing his sentence, Galileo dutifully knelt before the 10 judges and recanted his sin of promoting the Copernican system. One intriguing legend among the tales of scientific lore holds that, on rising from this humbling experience, the feisty old man flashed his still unbroken spirit by whispering, "*Eppur si muove*" ("Still it moves"), meaning, of course, Earth. It seems doubtful that one would take such a risk under the circumstances, but it certainly would have been in character for Italy's most famous pioneer of mathematics, physics, and astronomy.

According to historic record, the *segno* was revealed in a mode the Illuminati called *lingua pura*.

The pure language?

Yes.

Mathematics?

That's my guess. Galileo was a scientist after all, and he was writing *for* scientists. Math would be a logical language in which to lay out the clue.

—Dan Brown, *Angels & Demons*

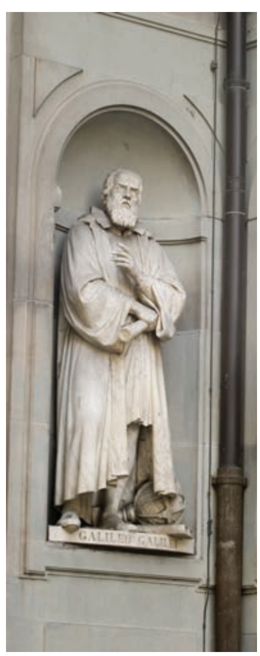


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FIGURE 4.3.7 Statue of Galileo in Florence, Italy.

Name ______________________

Exercise Set **4.3**

1. A ball dropped from a tall building takes 6.0 s to reach the ground (on Earth). What is the impact velocity of the ball, and how tall is the building?

2. A B1 bomber drops a bomb from a height of 6,000 m. Neglecting the effects of air resistance, how long does it take for the bomb to hit the ground, and what is its impact velocity?

3. Using Galileo's formulas for the velocity and distance of a dropped object on Earth, write the impact velocity v of the object as a function of the distance d it drops. (*Hint*: Solve the distance formula for t in terms of d, and substitute into the velocity formula.) What is the impact velocity of an object dropped from 100, 500, 1,000, and 4,000 m? Is this an increasing function?

Use these values to answer Exercises 4–9.

	Acceleration g (m/s²)
Mercury	3.8
Venus	8.9
Earth	9.8
Mars	3.7
Jupiter	24.9
Saturn	10.4
Uranus	8.8
Neptune	12.0
Pluto	0.7
Moon	1.6

4. In 10 s, how fast would an object dropped from rest be traveling on Venus? Mars? Neptune? How far would it drop on those three planets?

5. How long would it take a body to drop from a height of 500 m on Mercury? Earth? Saturn? What would the impact velocity be on those three planets? On which of these three planets would it take the least time to fall a specific distance? The greatest time?

6. Write the velocity v of the object as a function of the dropped distance d and the acceleration g. What is the impact velocity of a rock dropped from 100 m on Venus? Jupiter? Pluto?

7. Estimate a height from which you feel you could safely jump on Earth, and determine your impact velocity for that height. If you use that value as a safe impact velocity, from what height could you jump on Pluto and not be injured?

8. Lars weighs 890 N on Earth. What is his weight in pounds? What would he weigh (in pounds) on Venus? Mars? On which of the eight planets would his weight be a maximum?

9. Maria has a weight on Earth of 120 lb. What is her mass? What would her weight be (in pounds) on Mercury? Jupiter? Uranus?

10. Pounds are sometimes mistakenly used as units of mass. Use $W = 9.8m$ to show why the so-called conversion factor from kilograms to pounds is 2.2 lb/kg. Remember that first you must change pounds to newtons before you use this formula.

11. Luke Skywalker lands on the planet Xantok and measures the weight of a large igneous rock to be 533 N. Luke drops his light saber from the top of a cliff and determines the gravitational acceleration to be 6.5 m/s^2. What is the mass of the rock? What would it weigh on Earth in newtons? In pounds?

Assume each of the following statements is true. Write the converse and contrapositive of each statement, and classify each one as true or false.

12. If it is a cloudy day, then you do not catch fish on Fence Lake.

13. If someone is watching television, then the television is on.

14. If the light switch is up, then the light is on.

15. If a polygon is a square, then it is a rhombus.

16. If the wind is blowing, then the windmill is turning.

17. If a house is on Paradise Lane, then it costs at least $180,000.

18. If $x = 10$, then $x^2 = 100$.

19. If $x = 12$, then $2x + 15 = 39$.

20. If it does not rain, then we do not cancel soccer practice.

21. If my shirt is solid red, then it is not blue.

22. If the temperature is above 45°F, then it does not snow.

Determine whether each of the following five statements is true or false. Write the converse and contrapositive of each statement, and identify each one as true or false.

23. If $f(x)$ is an increasing function over $[a, b]$, then $f(a) < f(b)$.

24. If $f(x)$ is a decreasing function over $[a, b]$, then $f(a) < f(b)$.

25. If $f(x)$ is an increasing function, then $f(x)$ is a linear function with positive slope.

26. If $f(\theta) = \sin\theta$, then $f(30°) = 0.5$.

27. If $f(x) = 5x + 13$, then $f(x) = 23$.

28. Explain why Venus can be viewed going through a set of phases in the Copernican system but not in the Ptolemaic system. Do you think an outer planet (such as Mars) would exhibit two identifiably different sets of phases in the two systems? Explain.

29. Why didn't the observation of the phases of Venus prove the validity of the Copernican system? Why did it falsify the Ptolemaic system?

30. Galileo openly held a stronger opinion of the heliocentric model than did Copernicus. In what way? How did this position lead to conflict with the Catholic Church?

31. What book written by Galileo led to his being summoned to Rome to face the Inquisition?

32. According to legend, what did Galileo say upon recantation of his beliefs to the judges of the Inquisition?

33. Galileo offered a clever argument for why his formulas for the velocity and distance of a falling object were independent of mass. Suppose two objects x and y have different masses, with x being the lighter one. Aristotle claimed that y must fall faster than x. Galileo proposed, "What if we tie the two objects together and drop them?" Then the lighter object should retard the velocity of the combined x + y, and so the velocity of $x + y$ should be less than that of y alone. On the other hand, $x + y$ is more massive than y and so should fall faster, according to Aristotle. What is your conclusion to this thought experiment?

4.4 And All Was Light

Nature and Nature's laws lay hid in night:
God said, "Let Newton be!" and all was light.

—Alexander Pope

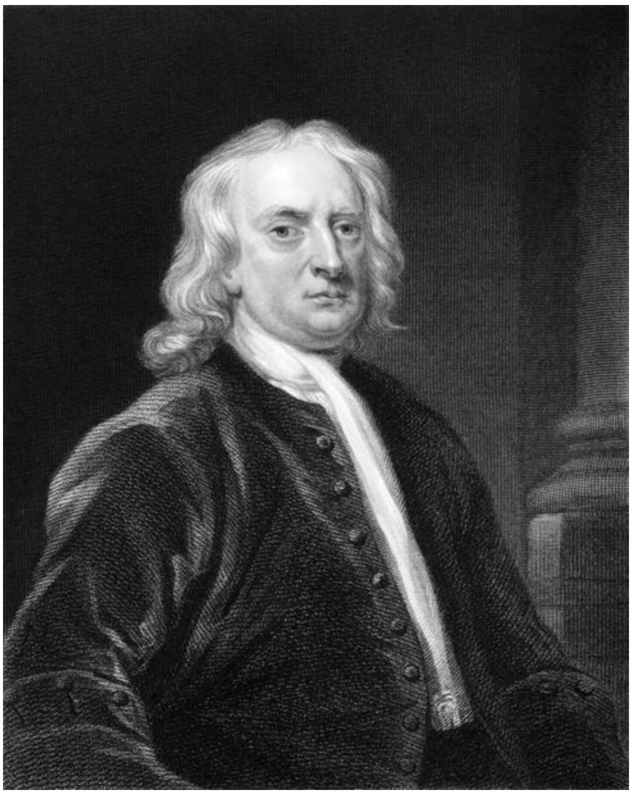

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FIGURE 4.4.1 Isaac Newton.

Galileo's sentence of life imprisonment was commuted to permanent confinement to his villa, where he spent the last nine years of his life studying physics and writing his finest book on the subject. As befits a good drama, 1642 saw both the death of Galileo and the birth of the man whom many declare to have possessed the most profound intellect in the history of humankind. **Sir Isaac Newton** (1642–1727) was a towering icon to the conquest of the mind over the mysterious universe (see **Figure 4.4.1**). He was described by biographer Gale E. Christianson as "a mutant, seeming more a phenomenon than a man . . . the incarnation of the abstracted thinking machine" and one who, in the words of Albert Einstein, "stands before us, strong, certain, and alone." An only child, born shortly after his father's death on a farm at Woolsthorpe, England, Newton experienced an unusually lonely childhood. By his third birthday, his mother had remarried an elderly rector to better her financial position, leaving young Isaac in the care of his aged grandmother at Woolsthorpe. As a youth, he spent many solitary hours constructing an array of mechanical devices such as water clocks, sundials, and various types of kites. He was not reunited with his mother until the age of 11, and the acute sense of abandonment felt by the young Isaac had a profound effect on his adult personality. A lifelong bachelor and recluse, he was penurious and sternly disciplined in his personal habits. Shunning the usual entertainments of civilized society, Newton was always happiest when left to himself to indulge his relentless curiosity. When asked later in life by one of his few friends, the astronomer **Edmond Halley** (1656–1743), how he had made his great discoveries, he responded, "By thinking about the problem unceasingly."

In 1665, Newton received his B.A. from Trinity College, Cambridge, where he had studied mathematics and physics. That same summer, the college was evacuated due to the spread of the bubonic plague that was devastating England and Europe, and so Newton returned to Woolsthorpe, where he spent two years in solitary contemplation of mathematics, optics, astronomy, and mechanics. This was a period in his life often referred to by historians as the *miracle years* because of the many discoveries he made at that time. In a 1718 letter, Newton wrote:

> In the beginning of the year 1665 I found the Method of approximating series . . . in May I found the method of Tangents . . . & in November had the direct method of fluxions . . . in January the Theory of Colours. . . . I began to think of gravity extending to y^e orb of the Moon & . . . from Kepler's rule . . . I deduced that the forces [which] keep the Planets in their Orbs must [be] reciprocally as the squares of their distances from the centers about [which] they revolve: & thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the earth, & found them answer pretty nearly. All this was in the two plague years of 1665 & 1666. For in those days I was in the prime of my age for invention & minded Mathematics & Philosophy [science] more then at any time since.

Here we see a mind overflowing with ideas. His "method of fluxions" led directly to his later development of the calculus, his "theory of colours" was the first comprehensive treatment of the properties of light, and his deductions about the force of gravity were the roots of the monumental treatise on the physics of motion he produced 20 years later. Newton states that the forces that "keep the Planets in their Orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve." This means that the gravitational force of the sun on any planet varies inversely as the square of the distance of the planet from the sun. Recall that if x and y are two variables and *y varies inversely as x,* or *y is inversely proportional to x*, then $y = k/x$, where k is a constant. Therefore, in this case, we may write $F = k/r^2$, where F is the force of gravity due to the sun and r is the planet–sun distance. Several other natural phenomena that involve varying quantities are known to behave according to this famous **inverse-square law**.

inverse-square law A general principle by which the effect of a phenomenon on an object varies inversely as the square of its distance to the source.

Think of a slowly inflating blue balloon. The dye giving the balloon its color keeps fading to lighter shades of blue as the balloon gets bigger. (See **Figure 4.4.2**.) This is so because a fixed amount of the dye keeps getting spread over a steadily increasing surface area. The surface area of a sphere of radius r is $4\pi r^2$. Assume the balloon to be a sphere painted evenly with b units of blue dye. There would be a concentration $C_0 = \dfrac{b}{4\pi R^2}$ units per square inch (in.^{2}) on the balloon's surface when it is inflated to a radius of $r = R$ in. If you blow up the balloon and double its radius to $r = 2R$, the concentration must decrease to $\dfrac{b}{4\pi(2R)^2} = \dfrac{1}{4}\left(\dfrac{b}{4\pi R^2}\right) = \dfrac{1}{4}C_0$, or one-fourth the original concentration. Similarly, if you tripled the original radius to $r = 3R$, the concentration would decrease to $\dfrac{b}{4\pi(3R)^2} = \dfrac{1}{9}\left(\dfrac{b}{4\pi R^2}\right) = \dfrac{1}{9}C_0$, and so on. We see that b and 4π are constants that play no part in the basic relationship between the concentration C and the radius r, which we can express as $C = k/r^2$, where $k = b/4\pi$.

If we think of light from a star as radiating from a single point source, we may imagine it to be traveling in spherical waves much as the ripples in a pond caused by a thrown stone emanate in circular waves from the point where the stone entered the water. Other phenomena such as gravity or radio waves can be visualized in the same way. As the light

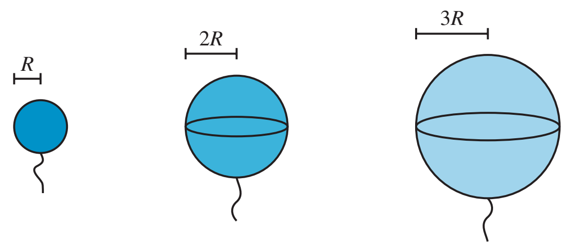


FIGURE 4.4.2 Demonstration of the inverse-square law.

gets farther from its source, it is spread out over a larger sphere. Hence its intensity I, like the concentration of dye on the balloon, also decreases inversely as the square of its distance r from the star. We may write

$$I = \frac{k}{r^2}.$$

Example 1

Suppose two stars, call them A and B, each have the same intrinsic brightness (like two lightbulbs of the same wattage). If star A is 20 pc away and the light intensity from star A is 25 times greater than that from star B, then how distant is star B? As with the balloon analogy, the sphere of light reaching us from B must have a larger radius than that of the sphere of light from A. The question is, by what factor? If I_A and I_B are the intensities of light from stars A and B, respectively, then $I_A = 25I_B$ and so

$$25 = \frac{I_A}{I_B} = \frac{k/20^2}{k/r^2} = \left(\frac{k}{20^2}\right)\left(\frac{r^2}{k}\right) = \frac{r^2}{20^2} = \left(\frac{r}{20}\right)^2.$$

The constant k cancels, and we equate the first and last terms to get

$$\left(\frac{r}{20}\right)^2 = 25$$
$$\frac{r}{20} = 5$$
$$r = 5(20) = 100\,\text{pc}.$$

Note that the desired factor turned out to be $\sqrt{25} = 5$. If, instead, we had received 25 times more light from star B than star A, then it would have to be *closer* by a factor of 5. This is so because $I_B = 25I_A$ in this case, and so $\frac{I_A}{I_B} = \frac{1}{25}$. Then the distance to star B would be found by

$$\left(\frac{r}{20}\right)^2 = \frac{1}{25}$$
$$\frac{r}{20} = \frac{1}{5}$$
$$r = 4\,\text{pc}. \quad \blacklozenge$$

It is worth stating the proportion discovered in this example involving the ratio of the distances of two stars and the ratio of their intensities. If I_A and I_B are the intensities of two stars A and B at distances r_A and r_B from Earth, respectively, then

$$\frac{I_A}{I_B} = \left(\frac{r_B}{r_A}\right)^2.$$

It was at Woolsthorpe where the famous tale of the falling apple allegedly occurred. In his old age, Newton was fond of reminiscing about it, yet no one is sure whether it actually happened or was a bit of mischief he concocted for the sake of a good story. Supposedly, he was musing in his garden one day, thinking deeply about the force that held the moon in its orbit, when he saw an apple fall from a tree. In a creative flash, he linked these two seemingly unrelated occurrences—the *same* power of gravity attracted the apple and kept the moon in its orbit—and unified the theories of terrestrial and celestial motion (See **Figure 4.4.3**.). Furthermore, he realized that *any* body exerts a gravitational force on any other body, which is a function of the distance between their centers, according to the inverse-square law. Key to this understanding was his formulation of the *central force law.* (See Exercise 26.)

FIGURE 4.4.3 Newton under the apple tree.

central force The force on a moving body directed toward a central fixed point.

centripetal acceleration The acceleration on a moving body directed toward the center of its path of motion that results from a central force.

Suppose you are holding a rope with a tether ball attached to the end, and you are rotating in a circle. You are providing a **central force** on the ball that imparts an acceleration on the ball that is directed toward you, the center. (Remember that acceleration has a directional component as well as a numerical component.) This force is necessary to keep the ball in a circular orbit around you. If the rope were to break, for example, the ball would go flying off in a straight line tangent to its previous circle of motion. Newton's central force law states that the acceleration a directed toward the center—known as **centripetal acceleration**—needed to keep an object in a circular orbit is

$$a = \frac{v^2}{r},$$

where v is the velocity of the object and r is its distance from the center. Note that this indicates that the acceleration increases as the distance decreases. You are familiar with this from riding in the passenger side of a car. If you are making a gentle left turn (large r) on the freeway, then little force (hence acceleration) is exerted by the seat and side of the car to keep you from going straight forward. However, if you try a very sharp turn (small r) at the same speed, you experience a much greater force—and you hope your car door is shut tight!

Example 2

Like the planets, the orbit of the moon has a low eccentricity, and so we may assume that it is circular for the sake of approximation. The radius of its orbit is $r = 3.84 \times 10^8$ m, and its period is $p = 27.3$ days = (27.3 days)(24 h/day)(60 min/h)(60 s/min) = 2.36×10^6 s. Recall that the Copernican velocity v is the circumference of its orbit divided by its period:

$$v = \frac{2\pi r}{p} = \frac{2\pi\left(3.84 \times 10^8\,\text{m}\right)}{2.36 \times 10^6\,\text{s}} = 1{,}020\ \text{m/s}.$$

The moon is under the influence of a central force—Earth's gravity. Its centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(1{,}020\ \text{m/s})^2}{3.84 \times 10^3\,\text{m}} = 0.0027\ \text{m/s}^2. \quad \blacklozenge$$

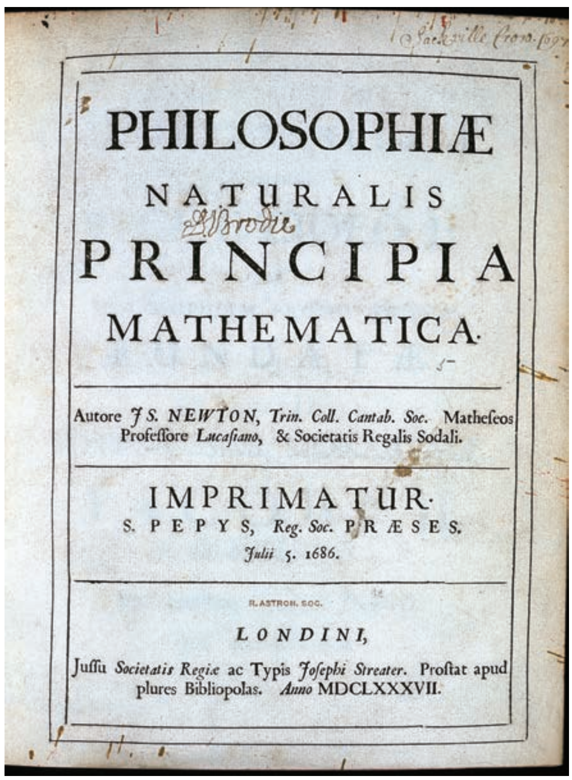
When Newton returned to Trinity at the age of 26, the results of his independent studies earned him the appointment to the prestigious position of Lucasian Professor of Mathematics at Cambridge. This position afforded Newton the freedom necessary for a life devoted to investigation and discovery. We can learn much from his methods. True, his genius is undeniable, but the many successes of Isaac Newton can also be attributed to his unwavering devotion to learning, his respect for the works of his predecessors, and, most importantly, his ability to learn from his mistakes.

Interestingly enough, Newton's greatest contribution may never have surfaced if not for a chance visit from a friend. In 1684, Edmond Halley came to Cambridge to see Newton and to pose a question that had been plaguing many of London's scientists: If the sun attracted each planet with a force inversely proportional to the square of the distance, what type of path would be traced out by the planet? Newton immediately replied that he had solved the problem long ago—it must be an ellipse. Astonished, Halley urged his friend to overcome his reluctance to publish and share this grand discovery with the rest of the world! So for the next 18 months, Newton brought his prodigious powers of

Principia The grand treatise of Isaac Newton that established the foundations of physics and astronomy. Its explanations for natural phenomena are mathematically derived from a core set of three laws of motion.

reasoning and insight to bear on finishing his greatest masterpiece, *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*). The magnificent **Principia**, as it is known, is considered to be one of the supreme achievements of the human mind (see **Figure 4.4.4**). In the beginning, he states his goal:

> For the whole burden of philosophy seems to consist of this—from the phenomena of motions to investigate the forces of nature and then from these forces to demonstrate all other phenomena.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

R. ASTRON. SOC.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

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FIGURE 4.4.4 The *Principia*.

laws of motion The three basic principles of motion, which Newton states in the beginning of the *Principia*, and from which he derives the causes for many celestial and terrestrial phenomena. See Figure 4.4.5.

force An entity applied to an object, that changes its velocity. It is equal to the product of the mass of the object and its acceleration.

inertia The ability of a body to resist a change in its state of motion.

He then states his famous three **laws of motion** (**Figure 4.4.5**), axioms that establish the foundations for a wealth of results concerning mathematics, physics, and astronomy. The second law is a means of recognizing when a **force** has been applied to an object: either the speed or the direction of motion changes (i.e., an acceleration has been added). **Inertia** is the natural resistance to such changes, and so this rule is often referred to as the *law of inertia.* The path of a thrown baseball is a long, graceful arc because the force of gravity continuously acts on the ball to overcome its inertia, serving to decrease its speed and pull it downward. If it were thrown in the negligible gravity of outer space, the ball would continue in a straight line forever. This explains why a space vehicle launched from Earth can travel such long distances with so little fuel. Once free of the gravitational field of Earth, it travels in a near-linear path (affected only by the sun's gravity) until the firing of a small steering rocket redirects it or it enters the gravitational field of another planet.

NEWTON'S LAWS OF MOTION

1. The Law of Inertia
   A body remains at rest or continues in a straight line at a constant speed unless acted upon by an external force.
2. The Law of Force
   The total force on a body is equal to the product of the mass of the body and its acceleration.
3. The Law of Equilibrium
   If one body exerts a force on a second body, then second body must exert an equal and opposite force on the first body.

FIGURE 4.4.5 The simplicity of these natural laws is beautiful.

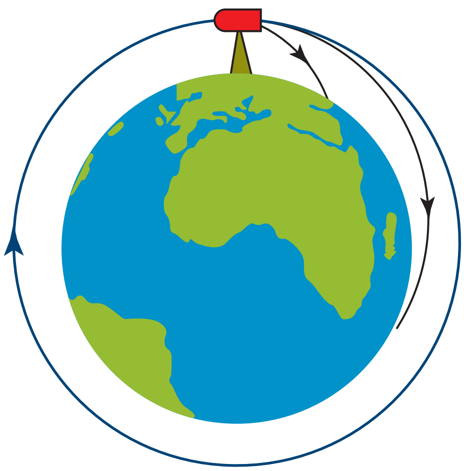


FIGURE 4.4.6 Newton's idea for putting a cannonball into orbit.

The first law allowed Newton to understand how Earth's gravity keeps the moon in orbit around it and, similarly, how the sun's gravity is responsible for the orbits of the planets. He compared the process to the firing of a cannonball off the top of a tall mountain (see **Figure 4.4.6**). As with a thrown baseball, Earth's gravitational force imparts an acceleration that continuously pulls the cannonball toward the surface. The greater its initial velocity, the farther it travels before impact. Surely, then, if it is endowed with a large enough initial velocity, its curved path keeps "missing" Earth, and so it enters a perpetual orbit as the pull of gravity and the inertia of the cannonball achieve a sort of balance.

In Example 2, we saw that the centripetal acceleration of the moon is 0.0027 m/s^2. Just as the twirling tetherball is pulled by the rope toward you, the moon is pulled by Earth's gravity with enough force every second to impart to it a speed of 0.00271 m/s toward Earth's center. Combined with its existing inertial velocity, this is just enough pull to keep it in orbit, as we see in **Figure 4.4.7(a)**. A schematic diagram in part **(b)** of the figure shows how the moon moves to the corner of the rectangle formed by the inertial velocity and the velocity change induced each second by the centripetal acceleration toward Earth. We hasten to add that because this is a continuous process, the end result is a smooth curve, not a sequence of jumps.

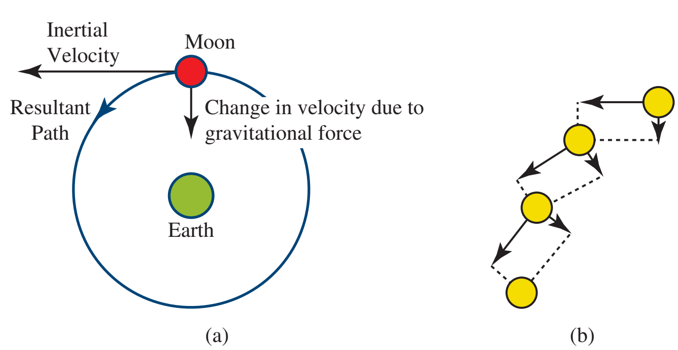


FIGURE 4.4.7 The orbit of the moon.

Newton was now in a position to test his theory from Woolsthorpe concerning gravitational force. Galileo had shown the value of acceleration due to gravity at Earth's surface (1 radius from its center) to be 9.8 m/s^2. The distance to the moon had been refined since the days of Aristarchus to be about 60 Earth radii. If the force of gravity (and its imparted acceleration to an object) *did* reach out into space as far as the moon and was inversely proportional to the square of the distance, its value there should be $\left(\frac{1}{60}\right)^2 = \frac{1}{3,600}$ of that at the surface. Lo and behold, $\frac{9.8}{3,600} = 0.0027\ \text{m/s}^2$, beautifully matching his previous computation or, in Newton's words, "found them answer pretty nearly." This was the validation he needed to confirm the inverse-square relationship.

Newton's second law of motion is formulated simply as

$$F = ma.$$

Perhaps an easier way to grasp its meaning is to write it as $a = \frac{F}{m}$. If we deliver a push to a tennis ball on a flat table with the same force as to a bowling ball, we know that the bowling ball will acquire less acceleration than the tennis ball because the bowling ball has the greater mass. Recall our previous examination of the relationship between mass and weight. This is seen now simply as a special case of this law, with force F replaced by weight W and a replaced by the acceleration g induced on the surface of a planet by gravity.

The third law can best be illustrated by imagining yourself in *Le Shrimp*, the smaller of two boats next to each other on a lake. If you give a forceful shove to *Goliath*, the larger boat, you will simultaneously experience a force of equal value returned to you. Both boats will accelerate—but in opposite directions. Again by applying the second law, *Le Shrimp* will achieve a greater velocity a few seconds later than *Goliath* because of its smaller mass. (See **Figure 4.4.8**.)

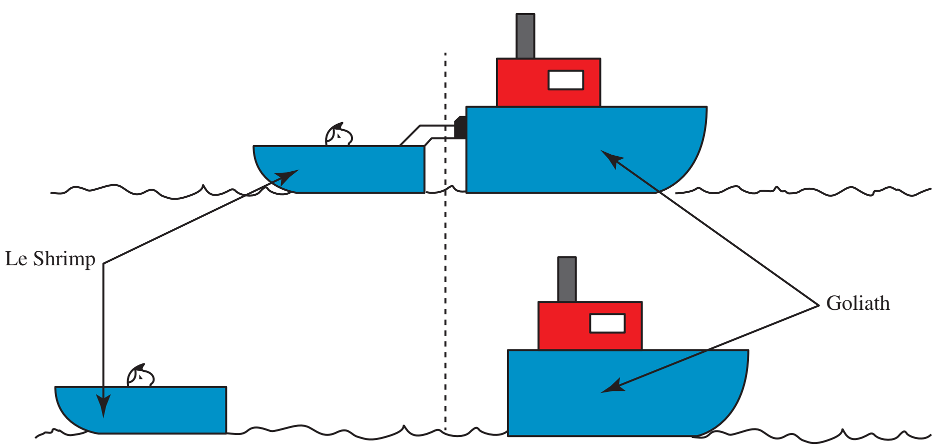


FIGURE 4.4.8 The second and third laws of motion. The small boat receives an equal and opposite force to the large boat but with a greater acceleration because of its smaller mass.

Law of Universal Gravitation Any body attracts another body with a force directed along the line joining their centers, which is proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.

These two laws led Newton to his final formulation of the crown jewel of his *Principia*, the **Law of Universal Gravitation**. If the force of body A on body B is to be equal (but in the opposite direction) to the force of body B on body A (third law) *and* each of these forces must include the mass of the corresponding body as a factor (second law), then the following statement must be true:

Every body in the universe attracts every other body with a force directed along the line joining their centers, which is proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.

In symbols, the force F is computed by

$$F = \frac{GMm}{r^2},$$

where M and m are the masses of the two bodies, r is the distance between their centers, and G is the constant of variation known as the *gravitational constant*. In 1798, English physicist Henry Cavendish made the first fairly accurate determination of this constant. In the metric system, its value is $G = 6.67 \times 10^{-11}$ newton-square meters per square kilogram ($N{\cdot}m^2/kg^2$).

Note that the force of gravity is a decreasing function of distance and an increasing function of mass. Although a space vehicle leaving our home planet frees itself of Earth's gravity after a relatively short distance, it continues to be under the influence of the sun for a considerably longer interval because the mass of the sun is about 330,000 times the mass of Earth.

So both mass and distance are important when computing the force of gravity of one body on another. If the mass of the moon were to somehow double, for instance, the resulting force between the moon and Earth would also double. On the other hand, because gravity follows the inverse-square law, if the distance from Earth's center to the moon or to any other orbiting satellite were to double, the gravitational force would decrease to one-fourth of the original value.

Next we observe that although gravitational force depends on mass, Galileo demonstrated that all bodies, regardless of mass, are attracted with the same acceleration to the surface of Earth. Newton's laws bear this out. Let M be the mass of Earth; m, the mass of any other body; and r, the distance between their centers. Because the force felt by the other body must be given by both of the force laws, we may equate the two expressions for F to get

$$ma = \frac{GMm}{r^2}$$

$$a = \frac{GM}{r^2}.$$

This formula for the acceleration of the body depends not on its own mass, but only on the mass of Earth and the other body's distance from Earth's center. Recall that this was precisely as Galileo had originally asserted!

Example 3

The previous argument shows us that the acceleration imparted by any central mass M to any body at a distance r is dependent *only* on r, according to the inverse-square relationship. Moreover, we need not know the value of M to compare the accelerations imparted at different distances. We proceed with the same method we used in comparing the intensity of light from two sources. For example, let a_E and a_J be the mean accelerations imparted by the sun to Earth and Jupiter, respectively, at their mean distances r_E and r_J from the sun. Then we have

$$\frac{a_J}{a_E} = \frac{GM/r_J^2}{GM/r_E^2} = \left(\frac{GM}{r_J^2}\right)\left(\frac{r_E^2}{GM}\right) = \frac{r_E^2}{r_J^2} = \left(\frac{r_E}{r_J}\right)^2.$$

Notice the similarity to our previous formula for the ratio of light intensities from two different stars. Because Jupiter is about 5.2 AU from the sun, $r_J = 5.2r_E$ and so

$$\frac{a_J}{a_E} = \left(\frac{r_E}{5.2r_E}\right)^2 = \left(\frac{1}{5.2}\right)^2 = 0.037.$$

Thus,

$$a_J = 0.037a_E.$$

So the mean acceleration imparted by the sun to Jupiter is 0.037 of that which it gives to Earth. If we conveniently define one gravitational unit (GU) to be the mean acceleration allotted to Earth by the sun, then Jupiter receives 0.037 GU. By the same token, at the closer mean distance of 0.39 AU, Mercury feels a greater pull from the sun than Earth by a factor of $1/(0.39)^2 \approx 6.6$. So we say that Mercury receives 6.6 GU from the sun. ♦

Example 4

When rocket scientists plan the launch of a satellite to orbit Earth, they make use of the relationship for acceleration a as a decreasing function of distance r from Earth's center in order to compute a at increasing distances. They already know by experiment that gravitational acceleration must equal 9.8 m/s^2 on the surface of Earth. This must therefore be the value for a at a distance from Earth's center equal to its radius, call it R. In other words, $a = 9.8$ for $r = R$. So we have

$$9.8 = \frac{GM}{R^2}.$$

Without substituting for any other letters, we can determine the acceleration induced by Earth at a distance of 2, 3, or 10 Earth radii. We simply let $r = 2R$ to get

$$a = \frac{GM}{(2R)^2} = \left(\frac{1}{2^2}\right)\left(\frac{GM}{R^2}\right) = \left(\frac{1}{4}\right)(9.8) = 2.45\,\text{m/s}^2.$$

Similarly, for $r = 3R$,

$$a = \frac{GM}{(3R)^2} = \left(\frac{1}{3^2}\right)\left(\frac{GM}{R^2}\right) = \left(\frac{1}{9}\right)(9.8) = 1.09\,\text{m/s}^2,$$

and for $r = 10R$,

$$a = \frac{GM}{(10R)^2} = \left(\frac{1}{10^2}\right)\left(\frac{GM}{R^2}\right) = \left(\frac{1}{100}\right)(9.8) = 0.098\,\text{m/s}^2.$$ ♦

Example 5

If we fix the distance in the prior formula, acceleration now becomes a function of mass. By substituting the radius of Earth (6,400 km), for r, 9.8 m/s^2 for a, and the value given earlier for the gravitational constant G, we can actually determine the mass of the Earth!

$$9.8 = \frac{\left(6.67 \times 10^{-11}\right)M}{\left(6.4 \times 10^{6}\right)^{2}}$$

$$M = \frac{(9.8)(6.4)^2 \times 10^{12}}{6.67 \times 10^{-11}}$$

$$\approx 60.2 \times 10^{23}\ \text{kg} = 6.02 \times 10^{24}\ \text{kg}.$$

(*Note*: Because the units for G are in terms of meters rather than kilometers, we needed to express the radius in terms of meters. Likewise, the units of mass are necessarily kilograms.) ♦

Because the acceleration induced by the sun is a decreasing function of distance, we must also note that this is consistent with our knowledge from a prior section that the maximum velocity of a planet occurs at perihelion and the minimum velocity at aphelion. And finally we consider how the special case of a circular orbit (constant r) of an object around the sun or any central body of mass M leads to the velocity function well known to every aerospace engineer. We have seen that the centripetal acceleration that causes the object to move in a circular motion of constant velocity v at a distance r is given by $\frac{v^2}{r}$. Equating this to the acceleration that the central body induces on the object, we get

$$\frac{v_c^2}{r} = \frac{GM}{r^2}$$

or

$$v_c = \sqrt{\frac{GM}{r}}.$$

circular velocity
Constant speed at which a body under the influence of a central gravitational force must move to maintain a circular path.

This value is often referred to as the **circular velocity** and is denoted by v_c.

Example 6

The first human-made satellite to be successfully placed in orbit around our planet was *Sputnik I*, launched by the Soviet Union on October 4, 1957. With what velocity did it move through its circular orbit? Because its orbit only carried it a few hundred miles above the surface, we use a value for r that is approximately equal to Earth's radius. Using a slightly more accurate value for the mass of Earth than computed in Example 5, we get

$$v_c = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{6.4\times10^{6}}}$$
$$= 7{,}900\ \text{m/s} = 7.9\ \text{km/s}.$$

(Getting beat into space by our cold war opponents had an electrifying effect on the United States. Fear of being "bombed from above" by Soviet space vehicles triggered a funneling of funds into space research and spurred reforms in mathematics and science education.) ♦

Example 7

Because the mass of the sun is 2.0×10^{30} kg, the speed of any object in a circular orbit at a distance of 1.5×10^{11} m (Earth's distance) is

$$v_c = \sqrt{\frac{(6.67\times10^{-11})(2.0\times10^{30})}{1.5\times10^{11}}} = 3.0\times10^{4}\ \text{m/s} = 30\ \text{km/s}.$$

You should compare this to the Copernican velocity you computed for Earth in Exercise Set 4.3. They match to two significant figures. The difference is that you need to have *observed* the movement of a known body in order to compute the Copernican velocity. For a given central mass, v_c is a function of r only. ♦

natural laws
Foundational principles that provide a basis of explanation of other relationships found in nature.

It is difficult to overestimate the impact of Newton and his *Principia* on the development of science and on the shape of modern times. His laws of motion and gravitation were the first set of **natural laws**—underpinning principles that provide a basis for the derivation of a host of other relationships. Following the statement of the laws in the book, he demonstrates how all three of Kepler's laws are immediate mathematical consequences. He then goes on to give the first mathematical treatment of wave motion; explains the orbits of comets; calculates the masses of the sun, Earth, and the planets with satellites; accounts for the equatorial bulge of Earth and how it causes the precession of the equinoxes; and shows how the gravitational pulls of the moon and sun are responsible for the daily high and low tides. Kepler and Galileo had obtained formulas in an empirical fashion; that is, they noticed mathematical patterns in the data that they encoded in equations. Although these equations were far better descriptions of nature than those that had been rendered by Aristotle, they failed to provide *reasons* for why they were true. Newton supplied this crucial missing piece of the puzzle. His laws were important because

1. They unified the laws of terrestrial and celestial motion.
2. They threw out the wordy, dead-end descriptions favored by Aristotelians and replaced them with crisp, simple axioms from which many observable phenomena could be deduced.
3. They were predictive. Positions of planets (and the paths of future space vehicles) could be calculated with great precision for the first time.

The last statement deserves a special mention. The predictive power of any new theory is one of the key tests of its acceptance by the scientific community. The many dramatic successes of Newton's calculus and natural laws brought quick and unanimous acknowledgment of the brilliance of his work. Mathematics became the new tool for the study of nature. Edmond Halley, for example, was able to employ the new mathematics to accurately forecast the return of the comet that bears his name. (See **Figure 4.4.9**.) It was not long before all serious doubters of the heliocentric theory disappeared. The revolution begun by Copernicus was complete.

FIGURE 4.4.9 Halley's Comet on a 1986 stamp from Laos.

Newton was lionized in his day in recognition of his accomplishments. He reigned as president of the Royal Society for 20 years and was given the lucrative position of Master of the Mint of England, a service he performed so admirably that he was knighted by the queen. He did, however, suffer his share of personal misfortune. In 1693, he had an acute mental breakdown, probably due to the poisoning of his system from the chemicals he routinely tasted in his alchemy experiments. He recovered from this episode, but he was never able to master his lifelong paranoia and eccentric, reclusive behavior, as evidenced by his bitter 25-year quarrel with the great German mathematician Gottfried Leibniz over who first created the calculus.

So yes, Newton was a strange bird to be sure, but we can be sure of one thing—he revealed a grand order in the universe. Every time one of America's splendid space vehicles is launched or a new space communications satellite is put into orbit, the memories of Isaac Newton and those who preceded him are recalled. While reflecting on his career late in life, he credited his predecessors by remarking that

> If I have seen farther than other men,
> it is because I stood on the shoulders of giants.

The stories of these giants—Copernicus, Kepler, Galileo, and Newton—offer a rich historical lesson. Cosmological conjecturing has occupied, and always will occupy, a special niche in the human quest for intellectual fulfillment. Although the search for solutions to the many puzzles offered up by Nature cannot be wholly separated from the surrounding social structure, it is, at its core, an individual endeavor. Any *absolute truth* as to the fabric of the universe can never be fully realized, only glimpsed from afar. It is those glimpses, those momentary clear visions in the mind of a single person—of you—that add an essential element to your life. It is incumbent upon you, therefore, to garner as much information as you can in order to experience the sharpest glimpses possible.

> Where the statue stood
> Of Newton with his prism and silent face,
> The marble index of a mind for ever
> Voyaging through strange seas of Thought, alone.
>
> —William Wordsworth

Name ______________________

Exercise Set **4.4**

1. Where was Isaac Newton born? What prolonged event prompted him to return there in 1665 for two years of isolated study?

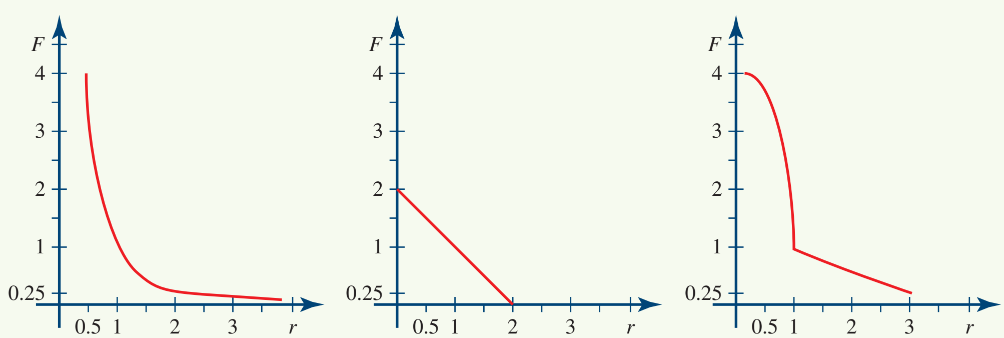
2. Who prompted Newton to publish his results in what eventually became the masterpiece, *Philosophiae Naturalis Principia Mathematica*?

3. Star A and star B have the same intrinsic brightness. If star A is 15 pc away and we receive 9 times as much light from A as from B, then how far away is star B? If we instead receive 9 times as much light from star B as from star A, how far away is it?

4. Neptune lies at an average distance from the sun of 30 AU. What is the intensity of light from the sun on Neptune as a portion of the intensity on Earth? Mercury lies at an average distance from the sun of 0.4 AU. By what factor is the intensity of light from the sun on Mercury greater than the intensity on Earth?

5. Stars A_1 and A_2 have the same intrinsic brightness and lie at distances of 14 and 56 ly, respectively. How much more light do we receive from A_1 than from A_2?

6. Which of the following is representative of the graph of gravitational force *F* between two given bodies as a function of distance *r*? Note that specific units are not necessary. Just think of one force unit existing at one distance unit. Is *F* an increasing or a decreasing function of *r*?



7. If the mass of Earth were magically doubled, how would the resulting gravitational force on any orbiting satellite change?

8. If the distance between Earth and the moon were doubled, what fraction of the original gravitational force between them would remain in effect?

9. If we define 1 gravitational unit (1 GU) to be the mean acceleration due to gravity that the sun imparts to Earth, how many gravitational units does the sun exert on Venus? Mars? Uranus? Pluto? The asteroids in the asteroid belt?

Body	**Mean Distance from Sun (AU)**
Venus	0.723
Mars	1.524
Asteroids	2.77 average
Uranus	19.18
Pluto	39.44

10. The diameters of Venus and Earth are almost identical. What must account for the fact that the surface gravity acceleration of Venus is less than that of Earth?

11. The acceleration on the surface of Mars is 3.7 m/s^2. What acceleration does it induce at a distance from its center of 2 Mars radii? 5 radii? 8 radii?

The gravitational constant $G = 6.67 \times 10^{-11}$ N·m^2/kg^2 will be needed for Exercises 12–17.

12. You land on the moon and you wish to determine its mass. You already know its radius is 1,740 km, and you measure the gravitational acceleration near its surface to be 1.6 m/s^2. What is the mass (in kilograms) of the moon?

13. The mass of Uranus is 8.68×10^{25} kg, about 20 times greater than the mass of Venus at 4.87×10^{24} kg. However, the gravitational accelerations at the surface of both planets are about the same. Why? Compute the radius of each planet, given the surface accelerations of Venus and Uranus of 8.9 and 8.8 m/s^2, respectively. Are the values for the radii consistent with your response?

14. A comet is a mixture of ice and carbonaceous dust often referred to as a dirty snowball. Comets in our solar system typically have very eccentric orbits. Halley's comet achieves perihelion at 0.53 AU from the sun and aphelion at 35.1 AU. Determine the acceleration (in meters per square second) of Halley's comet toward the sun at perihelion and aphelion. (The mass of the sun is 1.99×10^{30} kg.)

15. The mean distance of the moon from Earth is $r = 3.84 \times 10^8$ m. At that distance, we saw in Example 2 that the centripetal acceleration needed to keep the moon in orbit is $a = 0.0027$ m/s^2. Determine the mass of Earth by using these values in the relationship $a = \frac{GM}{r^2}$, which gives the acceleration imparted by Earth at that distance. Does this value match our previously computed value of $M = 6.02 \times 10^{24}$ kg?

16. We can determine the mass M of the sun in a manner similar to the previous exercise. We equate the centripetal acceleration needed to keep Earth in its orbit ($r = 1.5 \times 10^{11}$ m) to the acceleration imparted by the sun at that distance and solve for M.

$$\frac{v^2}{r} = \frac{GM}{r^2},$$

$$M = \frac{v^2 r}{G}.$$

Use the Copernican velocity of Earth $v = 3.0 \times 10^4$ m/s, which we already computed in problem 7 of Exercise Set 4.3 to find the mass of the sun.

17. The immense gravitational hold of the massive Jupiter (1.9×10^{27} kg) on the four Galilean moons has forced them into almost perfect circular orbits. Because any satellite of a planet must also behave according to Newton's laws, use this table to answer the following questions.

Moon	Distance from Jupiter (km)
Io	422,000
Europa	671,000
Ganymede	1,070,000
Callisto	1,883,000

(a) Which moon must have the greatest circular speed?
(b) Which moon must have the smallest circular speed?
(c) What is the circular speed of Europa? (Remember to use meters as the units for *r*.)

18. When *M* stands for the mass of Earth, $GM = 4.0 \times 10^{14}$ and so the circular velocity v_c of a satellite orbiting Earth reduces to

$$v_c = \sqrt{\frac{GM}{r}} = \frac{2.0 \times 10^7}{\sqrt{r}}. \quad \text{(Remember, } r \text{ must be in meters.)}$$

How fast must a satellite travel to remain in a circular orbit around Earth with an orbital radius of 9,000 km from Earth's center?

19. One of the most practical applications of Newton's discoveries in modern times is the use of communication satellites. These satellites are placed in strategic orbits to relay radio signals between two distant locations on Earth. A *geostationary* satellite is one that always remains above the same place on Earth's surface, and so its orbital period must be 24 h. The previous exercise gives the velocity v_c necessary for a circular orbit around Earth as a function of *r* alone. Determine the radius *r* necessary for the orbit of a geostationary satellite by equating this formula to $\frac{2\pi r}{24(60)(60)}$ and solving for *r*.

20. The electrostatic force acting between two charges also follows the inverse-square law. In 1780, Charles Augustin de Coulomb showed that the force is proportional to the product of the two charges and inversely proportional to the square of the distance between them. Letting *E* stand for the electrostatic force, *Q* and *q* for the two charges, and *r* for the distance, write the function for *E* in symbols. (See the statement of the Law of Universal Gravitation.)

21. The force between two charges is 0.0045 N. Using the function for *E* given in the previous exercise, what is the new force if the distance between these two charges is increased by a factor of 3?

22. Suppose an object is at a distance *r* from the center of a body of mass *M*. The speed required for that object to escape from that central body's gravitational hold is called the *escape velocity* v_e and is given by the function

$$v_e = \sqrt{\frac{2GM}{r}}.$$

Note the similarity to the function that gives the circular velocity v_c. If an object were already in a circular orbit around a central mass, by what factor would it have to increase its velocity in order to escape?

23. What are the advantages of Newton's laws of motion as a set of axioms to describe nature over the explanations offered by Aristotle?

24. For a body in a circular orbit around the sun, we have two algebraic expressions that give us the speed of the body: $v_c = \sqrt{\frac{GM}{r}}$ and the Copernican velocity $v = \frac{2\pi a}{p}$ from the last section. Replace r with a, equate these two expressions, and deduce Kepler's third law.

25. In Example 5, we determined the mass of Earth by knowing the value of the acceleration it produced on a body near its surface. In other words, we knew the value of acceleration that Earth imparts to an object at a specific distance. Describe a procedure by which we could determine the mass of a planet possessing moons by observing the radius and period of the orbit of one moon.

26. Kepler's third law was the key to Newton's realization that the gravitational force of the sun on the planets followed an inverse-square law. The *central force law* states that the centripetal acceleration a imparted by the sun on a body in a circular orbit of radius r moving with velocity v is given by

$$a = \frac{v^2}{r}.$$

According to Kepler, $p^2 = kr^3$, where p is the period of revolution of a body. This means that the ratio $\frac{r^3}{p^2}$ is always constant. Substitute the Copernican velocity $v = \frac{2\pi r}{p}$ for v to show that the acceleration a (and hence the force) varies inversely as the square of the distance r.

27. We think of the orbit of the moon as resulting solely from Earth's gravitational pull, but in fact the sun also exerts a significant influence because of its enormous mass. This is true for a satellite of any planet. In the May 1963 issue of *The Magazine of Fantasy and Science Fiction*, Isaac Azimov computed what he called the "tug-of-war" value. For any satellite of mass m, he defined this to be the ratio of F_p, the gravitational force of the planet, to F_s, the force due to the sun. Specifically,

$$\frac{F_p}{F_s} = \frac{GM_p m/r_p^2}{GM_s m/r_s^2} = \left(\frac{M_p}{M_s}\right)\left(\frac{r_s}{r_p}\right)^2,$$

where M_p and M_s are the masses of the central planet and sun, respectively, and r_p and r_s are the distances of the satellite from the planet and sun, respectively. The Galilean moons of Jupiter, for instance, have ratios ranging from a high of 3,260 for nearby Io to a low of 160 for the more distant Callisto. It turns out that our moon is the only major satellite in the solar system with a tug-of-war ratio less than 1. What does this mean? Rather than a satellite circling Earth, what might be a better description of the orbital path of the moon? Compute the tug-of-war ratio for the moon. (Recall from the first section that the sun is 390 times as far from Earth as the moon. Also, the mass of Earth is 0.000003 of the sun's mass.)

28. A *black hole* is an object in space containing a very large amount of mass in a volume so small that the resulting gravitational field is strong enough to prevent even light from escaping. Explain how this is consistent with the roles of force F and distance r as they are related in Newton's Law of Universal Gravitation.

Name ____________________

use the extra space to show your work

Chapter Review Test 4

1. What new idea was introduced by Heracleides to the planetary model concerning the orbits of Mercury and Venus?

2. Define an axiom, and explain the role of a set of axioms in creating a model of a natural phenomenon.

3. What star in the sky remains practically motionless throughout the night? Does it appear higher above the horizon as viewed from Scranton, Pennsylvania, or Atlanta, Georgia?

4. What is the mean distance of Earth from the sun, measured in light-seconds?

5. If you go outside and look at the constellation Orion at 10 P.M. one winter night and then again a month later at 10 P.M., about how many degrees has Orion shifted across the sky?

6. Saturn has an orbital period of 29.5 yr. Compute its mean distance (in astronomical units) from the sun and its Copernican velocity (in kilometers per second).

7. How far away is a star having a parallax angle of 0.28″? Give your answer in parsecs and light-years.

8. Halley's comet orbits the sun in an elliptical orbit, with a perihelion distance of 0.53 AU and an aphelion distance of 35.1 AU. Compute the eccentricity of its orbit. Would you say the shape of this orbit resembles a circle or a cigar?

9. Given that the eccentricity of Mercury is 0.2056 and its maximum distance from the sun is 0.467 AU, find its minimum distance from the sun.

10. Consider the statement "If it is a July day in Phoenix, then the temperature is over 100°." Write the converse and contrapositive of this statement. Assuming the original statement is true, is the converse necessarily true? Is the contrapostive necessarily true?

11. List two observations made by Galileo that convinced him of the validity of the Copernican system.

12. The acceleration due to gravity near the surface of Mars is 3.7 m/s^2. How much time would it take for an object to drop from 1,250.6 m on Mars? What would be the impact velocity?

13. Stars Alpha and Beta have the same intrinsic brightness. If Alpha lies at a distance of 50 pc from Earth and we receive $\frac{1}{49}$ as much light from Beta as from Alpha, then how far away is Beta?

14. Mercury and Mars have about the same surface acceleration due to gravity, yet Mars has a greater radius than Mercury. Use the law of universal gravitation to explain what must account for this fact.

15. The planet Uranus lies at a mean distance of about 19 AU from the sun. If 1 GU is the gravitational acceleration exerted by the sun on Earth, how many gravitational units does the sun exert on Uranus?

16. The asteroid belt is a region between the orbits of Mars and Jupiter in which thousands of chunks of rock known as asteroids revolve around the sun at an average distance of about 2.77 AU. Even though they have many different masses, is the acceleration imparted to them by the sun about the same for all of them? Why? What is it? (The mass of the sun is 1.99×10^{30} kg.)

17. You land on the moon, and you wish to determine its mass. You already know its radius is 1,740 km, and you measure the gravitational acceleration near its surface to be 1.6 m/s^2. What is the mass (in kilograms) of the moon?