ACTIVITY
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Eratosthenes Measures Earth

In trying to understand the cosmos, the ancient Greeks used geometry to dazzling effect. Around 200 BC, the astronomer Eratosthenes conceived a way to measure the size of Earth by simple observation. A brilliant polymath, Eratosthenes had left his native Cyrene, in present-day Libya, to study in Athens before moving to the intellectual hotbed of the Mediterranean: Alexandria, Egypt. He was appointed tutor to King Ptolemy III's son, Philopater, and eventually director of the famous Alexandrian Library.

Eratosthenes learned of a curious phenomenon that occurred once a year in the Egyptian village of Syene (pronounced sī-ē′nē, the present-day city of Aswan), now known to be some 500 miles south of Alexandria. At noon on the first day of summer, the Sun stood at the zenith—directly overhead—in Syene. A gnomon cast no shadow, and the Sun's rays shone straight down to the bottom of a deep well on nearby Elephantine island, illuminating the water below. Eratosthenes was intrigued by this report. He knew that on the same day in Alexandria, the noontime Sun did *not* stand at the zenith, but approximately *7 degrees* away. In Alexandria, a gnomon cast a shadow, and the Sun's oblique rays struck the wall, not the bottom, of the wells. From this simple observation, Eratosthenes computed the size of Earth, just as you are going to do now.

Dividing the Circle

First, some practice with circles and angles. Fact: A circle contains 360 degrees. Dividing a circle will yield pie-shaped sectors whose degree measures must add up to 360 degrees.

1. On the worksheet, draw a line dividing the illustrated circle in half. (See **Figure 6.1**.) What do you think is the degree measure of each sector? Now divide the circle so that it has four equal sectors. What do you think is the degree measure of each of these sectors? Divide the circle further into eight equal sectors. What do you think is the degree measure of each of these sectors?
2. Based on your answers to Part 1, write down a general rule specifying how to compute the degree measure of each sector if given the number of sectors in a circle.
3. Now write down the *reverse* rule, in other words, the rule specifying how to compute the number of sectors in a circle if given the degree measure of each sector.
4. Using your rule from Part 3, compute the number of sectors in a circle if each sector is 7 degrees wide. Round your answer to the nearest whole number. Circle your answer; you'll need it later.

Parallel Lines

In deducing the size of Earth, Eratosthenes had to make a critical assumption: that Earth is so small and so far away from the Sun that the Sun's rays are nearly parallel when they strike it. That is, the Sun's rays illuminate Alexandria and Syene from essentially the same direction. You can illustrate that this is true by drawing a scale model of the Earth–Sun system, where the size and spacing of the Sun and Earth have been uniformly shrunk until they "fit" down the side of your worksheet.

- Fact: The Sun's diameter is about 100 times Earth's diameter.
- Fact: Earth is roughly 100 Sun-diameters away from the Sun.

5. In the right-hand margin of the worksheet, at the top, draw a *small* circle to represent the Sun. From this "Sun-circle," draw a vertical line along the right-hand side of the worksheet that is 100 times your Sun-circle diameter. Remember, Earth on this scale will be 100 Sun-circles away, so your Sun-circle has to be small enough that 100 of them can fit in a line down the side of the page. Note: You do not need a ruler for this part; do your best to draw a string of 100 Sun-circles of the same diameter.
6. At the lower end of the line you just drew, you will attempt to draw a circle to represent Earth. But first, describe how big your "Earth-circle" would be, *to scale*, compared to your Sun-circle. Your description should be both quantitative and qualitative, that is, in terms of actual numbers and in terms of words. Can you depict the Earth-circle realistically in this reduced-scale drawing?

Look at your scale drawing. Imagine the Sun's rays radiating in all directions from its surface. Now imagine only those rays heading in Earth's direction. Observe how these rays are virtually parallel, regardless of where on Earth they strike. Eratosthenes was right: the Sun's rays illuminate Alexandria and Syene from essentially the same direction.

What Eratosthenes Saw

Figure 6.2 on the worksheet shows a cross section of Earth depicting Alexandria, where Eratosthenes lived and, about 500 miles southward, Syene, where the famous well was located. (The well is not drawn to scale, nor is the separation between Alexandria and Syene; both have been exaggerated for clarity.) On this figure, you will draw parallel lines representing the Sun's rays. Note: The rays you will draw here do *not* come from the Sun-circle you drew previously, but originate from an imaginary Sun somewhere far off the page.

7. On the first day of summer, the Sun's rays shone directly down the well in Syene. Draw a line to represent one of these rays, that is, a line that starts near the right-hand edge of the worksheet and extends straight down to the bottom of the well. Extend this line straight to Earth's center, labeled **C** in the figure. Note that this "ray" coincides with the vertical direction in Syene.
8. Draw a second line *parallel* to the line you just drew, again starting near the right-hand edge of the page, but this time extending to the location of Alexandria. Now draw a third line representing the vertical direction in Alexandria; this line should extend from Earth's center **C**, through Alexandria, to a point in space above Earth's surface. The angle between this line and the Sun's ray represents what Eratosthenes saw: in Alexandria, the Sun appeared 7 degrees away from the vertical, even while the Sun was reported to be precisely vertical in Syene. Eratosthenes realized that the difference in the Sun's sky position was due to Earth's curvature. Label the Alexandria angle with its degree measure: 7°.
9. Note the angle that is formed by the intersection of lines at Earth's center **C**. It's no coincidence that this angle is equivalent to the 7-degree angle at Alexandria; there is a geometric rule that explains this. (*When a line crosses a pair of parallel lines, the corresponding interior angles are equal.*) Label the angle at Earth's center with its degree measure: 7°.

Finishing Up

Now you can figure out the size of Earth just as Eratosthenes did.

10. Note that the angle you just labeled at Earth's center defines a 7-degree-wide sector of Earth's entire circumference. Each 7-degree sector like this one encloses a 500-mile arc on Earth's surface, the distance between Alexandria and Syene, according to modern measurement. To compute Earth's circumference, all you need to do is multiply 500 miles by the number of 7-degree sectors in a circle, which you already determined in Part 4. How many miles are in the circumference of Earth? What is the Earth's diameter? (Recall that the circumference of a circle is π times the circle's diameter.)

    While we are unsure whether Eratosthenes got the equivalent of the modern-day result for Earth's circumference—his measurement unit, the length of a stadium, has no agreed-upon conversion into miles—he nonetheless demonstrated the global reach of ancient Greek geometry, as well as the analytical power of the human mind.

Name Section Date

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For credit, you must show all your work.

1.

Number of Sectors	Degree Measure of Each Sector
2	
4	
8	

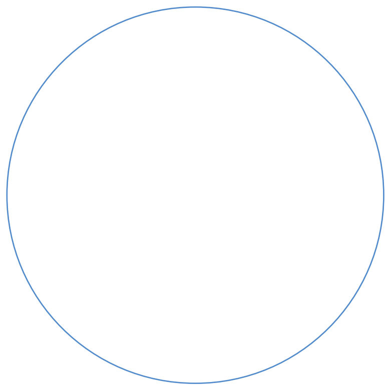

FIGURE 6.1

2. ____________________

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3. ____________________

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4. Number of 7-degree sectors in a circle (round off to the nearest whole number): ____________

5. Complete your scale drawing in the right-hand margin. (Refer to instructions.)

6. ____________________

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7, 8, 9.

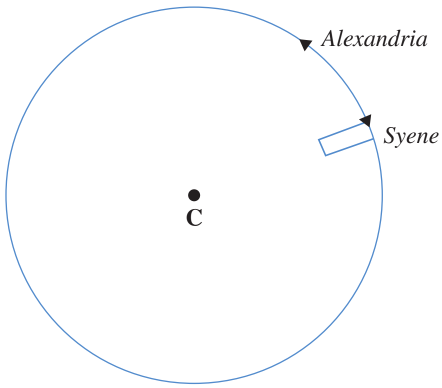

FIGURE 6.2 Cross-section of Earth showing Alexandria and the well at Syene.

10. Circumference of Earth: ________________ miles

Diameter of Earth: ________________ miles