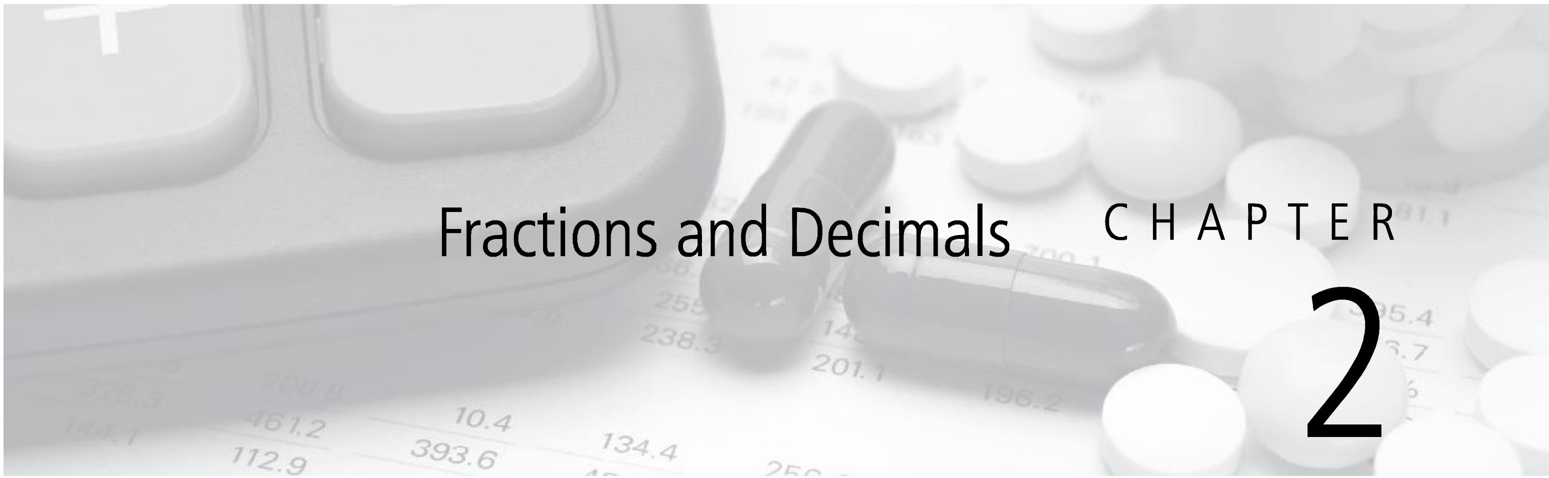

Fractions and Decimals

CHAPTER 2

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OBJECTIVES

- Interpret fractions
- Interpret decimals
- Add and subtract fractions
- Add and subtract decimals
- Multiply and divide fractions
- Multiply and divide decimals
- Determine the significant figures for decimals

FRACTIONS

A fraction represents a portion of a whole unit. Numerically, it is expressed as one number over another ($\frac{2}{6}$). The bottom number defines the number of parts it will take to make the whole unit. The top number defines how many parts of the whole there are. (See **Figure 2.1**.)

A fraction is made up of two components: the **numerator** and the **denominator**.

$\frac{2}{6}$ → numerator (2) → denominator (6)

2 → numerator
6 → denominator

The numerator defines the number of units in the whole you have or need. The denominator represents how many units make up the whole.

A fraction that has the same number in the numerator and the denominator equals one whole, meaning the number of units you have makes the whole complete. If you have 8 pieces of pie and the pie pan holds 8 pieces, then you have the whole pie.

A. $\frac{6}{6} = 1$ **B.** $\frac{7}{7} = 1$ **C.** $\frac{10}{10} = 1$

You can verify this by dividing the top number by the bottom number ($6 \div 6 = 1$). A number divided by itself is always one.

FIGURE 2.1 Fractions

A **proper fraction** is one that has a smaller number on top (numerator). This will always represent a number that is less than one.

A. $\frac{1}{3}$ **B.** $\frac{3}{7}$ **C.** $\frac{5}{6}$ **D.** $\frac{9}{12}$

When you divide the top number by the bottom number, you get a number less than one.

A. $1 \div 3 = 0.33$ **B.** $3 \div 7 = 0.43$ **C.** $5 \div 6 = 0.83$ **D.** $9 \div 12 = 0.75$

An **improper fraction** is a fraction that has a larger number on top (numerator). This number would represent a whole plus a portion of another whole.

A. $\frac{5}{4} = (1)\frac{4}{4} + \frac{1}{4}$ **B.** $\frac{8}{5} = (1)\frac{5}{5} + \frac{3}{5}$ **C.** $\frac{9}{7} = (1)\frac{7}{7} + \frac{2}{7}$

When you divide the top number by the bottom number, you get a whole plus a number less than one.

A. $5 \div 4 = 1.25$ or $1\frac{1}{4}$ **B.** $8 \div 5 = 1.6$ or $1\frac{3}{5}$ **C.** $9 \div 7 = 1.29$ or $1\frac{2}{7}$

A **mixed number** is a whole number with a fraction. You must make this number into an improper fraction in order to perform any calculations. To do this, you would multiply the bottom number in the fraction by the whole number, add the top number to that product, and then place that value in the numerator position over the original denominator.

$4\frac{3}{5} = 4 \times 5 = 20 + 3 = 23 \rightarrow \frac{23}{5}$; $2\frac{3}{8} = 2 \times 8 = 16 + 3 = \frac{19}{8}$

A **complex fraction** is a value that has a fraction in both the numerator and denominator position.

$\frac{\frac{1}{3}}{\frac{5}{8}}$; $\frac{\frac{2}{5}}{\frac{7}{10}}$; $\frac{\frac{3}{7}}{\frac{4}{5}}$

Addition and Subtraction

Adding and subtracting fractions requires a few rules in order to obtain the correct answer. When you add or subtract fractions, they must have the same denominator, or **common denominator**. Comparing "apples to apples" is another way to look at it. If you want to know how many apples you have, you can only add or subtract the apples. If you include the oranges, you will not get the correct answer.

To find a common denominator, you need to determine a number that both denominators have in common (common denominator). It could be one of the numbers in the fractions, or you may have to find **a** different number that is common to both.

EXAMPLE 1

$\frac{3}{12} + \frac{7}{12} = \frac{10}{12}$ $\qquad$ $\frac{11}{12} - \frac{4}{12} = \frac{7}{12}$

The denominators are the same (12), so you can just add or subtract the numerators and place them over the existing denominator.

EXAMPLE 2A

$\frac{2}{3} + \frac{1}{9} =$

The denominators are different (apples/oranges) so a common factor must be found.

Looking at these two numbers, we see that the 3 will go into the 9 (3 is a factor of 9). We need to rewrite the equation using 9 as the denominator for both fractions. You need to determine what must be done with the 3 in the first fraction to get it to equal 9. Whatever you do to the 3 (denominator), you must also do to the 2 (numerator) in order to keep the fraction equivalent.

$\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$

Now your equation looks like this:

$\frac{6}{9} + \frac{1}{9} = \frac{7}{9}$

EXAMPLE 2B

$\frac{4}{5} + \frac{3}{20} = \frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$

$\frac{16}{20} + \frac{3}{20} = \frac{19}{20}$

When it is not so obvious what the common denominator is, you need to follow a different strategy.

EXAMPLE 3A

$\frac{2}{15} + \frac{5}{6}$

You need to list multiples of each denominator to see if they have one in common.

15 = 15, **30**, 45, 60, 75
6 = 6, 12, 18, 24, **30**, 36, 42

The list shows that 30 is the first multiple that they share. The next step is to create your new equivalent fractions.

$\frac{2}{15} \times \frac{2}{2} = \frac{4}{30}$ $\quad$ and $\quad$ $\frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$

Your new equation looks like this: $\frac{4}{30} + \frac{25}{30} = \frac{29}{30}$

■ EXAMPLE 3B

$\frac{3}{8} + \frac{1}{6}$

8 = 8, 16, **24**, 32, 40
6 = 6, 12, 18, **24**, 30

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$ and $\frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$

$\frac{9}{24} + \frac{4}{24} = \frac{13}{24}$

Sometimes the only way to find a common denominator is to multiply them together because their product *is* the only thing common.

■ EXAMPLE 4A

$\frac{5}{9} - \frac{3}{7}$

Looking at these numbers you might assume that they do not have a common multiple other than their product (multiplying them).

Their common denominator is 7 × 9 = **63**.

$\frac{5}{9} \times \frac{7}{7} = \frac{35}{63}$ and $\frac{3}{7} \times \frac{9}{9} = \frac{27}{63}$

Your new equation is

$\frac{35}{63} - \frac{27}{63} = \frac{8}{63}$

If your equation has a mixed fraction, be sure to make it into an improper fraction, and then follow these steps to ensure common denominators are present.

■ EXAMPLE 4B

$1\frac{2}{3} + 2\frac{1}{9} =$

$(3 \times 1) + 2 = \frac{5}{3}$; $(9 \times 2) + 1 = \frac{19}{9}$

$\frac{5}{3} + \frac{19}{9} =$

$\frac{5}{3} \times \frac{3}{3} = \frac{15}{9} + \frac{19}{9} = \frac{34}{9}$

Convert to a mixed number:

$34 \div 9 = 3\frac{7}{9}$

Practice Problems 2.1

Simplify the answer to a proper fraction in its simplest terms.

1. $\frac{4}{5}+\frac{4}{5}=$
2. $\frac{3}{8}+\frac{2}{8}=$
3. $\frac{5}{12}+\frac{6}{24}=$
4. $\frac{1}{3}+\frac{4}{9}=$
5. $\frac{8}{27}+\frac{7}{8}=$
6. $\frac{9}{7}+\frac{2}{7}=$
7. $\frac{6}{7}-\frac{2}{7}=$
8. $\frac{7}{9}-\frac{1}{3}=$
9. $\frac{7}{15}+\frac{4}{60}=$
10. $\frac{3}{13}+\frac{4}{7}=$
11. $4\frac{3}{4}+\frac{7}{4}=$
12. $\frac{8}{5}+2\frac{1}{10}=$
13. $5\frac{2}{3}-2\frac{1}{3}=$
14. $\frac{12}{7}+\frac{3}{7}=$
15. $\frac{6}{21}+\frac{4}{21}=$

Multiplication and Division

Multiplying fractions is one of the simpler calculations to perform. You do not need to worry about common denominators. The numerators are multiplied together and then the denominators are multiplied together. The answer is then simplified to its lowest term.

EXAMPLE 5

$$\frac{3}{8}\times\frac{4}{6}=\frac{3\times 4}{8\times 6}=\frac{12}{48}\div\frac{12}{12}=\frac{1}{4}$$

Multiplying the numerators you get 12 and then multiplying the denominators equals 48. The fraction $\frac{12}{48}$ can be simplified to $\frac{1}{4}$ because 12 is a common factor in both the numerator and the denominator.

EXAMPLE 6

$$\frac{4}{6}\times\frac{2}{7}\times\frac{3}{5}=\frac{4\times 2\times 3}{6\times 7\times 5}=\frac{24}{210}\div\frac{2}{2}=\frac{12}{105}$$

Dividing fractions is done using multiplication also. When you divide two fractions, you will invert, or use the reciprocal, of the second fraction and then perform multiplication to solve the equation.

EXAMPLE 7A

$$\frac{6}{11}\div\frac{3}{8}=\frac{6\times 8}{11\times 3}=\frac{48}{33}=1\frac{15}{33}=1\frac{5}{11}$$

Solving this equation requires you to "flip" the second fraction and then multiply the numerators together followed by multiplying the denominators together. Simplifying the answer to its lowest terms makes it easier to read and determine its true value.

This answer is in the form of an improper fraction (the numerator is larger than the denominator). To make this a proper or mixed fraction, you need to pull out the "whole" number (1) and determine the remaining fraction. The "one" would be $\frac{33}{33}$. This leaves a remainder of 15 (48 − 33). Following the given equation, the 15 is then placed over the 33 once again. Looking at this fraction $\frac{15}{33}$, you notice that both numbers have a three (3) in common. Simplifying this to its lowest form means dividing both the numerator and the denominator by the common factor (3) to result in a fraction of $\frac{5}{11}$.

EXAMPLE 7B

$$\frac{7}{9} \div \frac{2}{5} = \frac{7}{9} \times \frac{5}{2} = \frac{35}{18} = 1\frac{17}{18}$$

EXAMPLE 8

Remember:
A fraction is simply another way to write a division problem!

$$\frac{8}{3/5} = \frac{8}{1} \times \frac{5}{3} = \frac{40}{3} = 13\frac{1}{3};\ \frac{\frac{2}{5}}{\frac{3}{7}} = \frac{2}{5} \times \frac{7}{3} = \frac{14}{15};\ \frac{\frac{2}{3}}{4} = \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

The same process is used if there is a fraction in the denominator of a fraction. Simply bring it out of the denominator position and invert it. Follow the multiplication process and reduce to lowest terms if necessary.

Practice Problems 2.2

Solve the following equations and simplify.

1. $\frac{5}{7} \times \frac{4}{8} =$
2. $\frac{2}{4} \times \frac{8}{13} =$
3. $\frac{12}{13} \times \frac{3}{5} =$
4. $\frac{3}{7} \times \frac{2}{9} \times \frac{1}{4} =$
5. $\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6} =$
6. $2\frac{3}{4} \times \frac{7}{8} \times 1\frac{1}{4} =$
7. $\frac{11}{12} \div \frac{3}{4} =$
8. $\frac{5}{6} \div \frac{1}{3} =$
9. $9 \div \frac{4}{5} =$
10. $\frac{1}{5} \div 6 =$
11. $3\frac{1}{2} \div \frac{7}{8} =$
12. $2\frac{3}{4} \div 1\frac{5}{8} =$
13. $\frac{3}{\frac{4}{5}} =$
14. $\frac{\frac{6}{7}}{\frac{1}{3}} =$
15. $\frac{\frac{3}{5}}{7} =$

DECIMALS

A decimal is another way to represent fractional parts of a whole unit. To convert a fraction to a decimal, you simply divide the numerator by the denominator.

EXAMPLE 9

$$\frac{3}{4} = 3 \div 4 = 0.75;\ \frac{2}{5} = 0.4;\ \frac{9}{5} = 1.8$$

Positionally, each digit to the right or the left of the decimal represents a fractional multiple of 10. The most common fractional units (right of the decimal) seen in pharmacy math are tenths (0.1), hundredths (0.01), and thousandths (0.001).

Many drugs are dosed in fractional units—less than one. When this occurs, it is important to utilize a leading zero in front of the decimal so that no mistake is made in interpreting the desired dose.

EXAMPLE 10

Levothyroxine 0.15 mg versus .15 mg
Digoxin 0.125 mg versus .125 mg

The levothyroxine could be misinterpreted as 15 mg if you could not see or read the decimal in front of the one. The same applies for the digoxin dose. The leading zero in these cases verifies that the dose is a very small number—less than one.

Addition and Subtraction

Addition and subtraction of decimals has one basic rule: *Always line up the decimal points when adding or subtracting.*

EXAMPLE 11

22.6	124.342	156.43
14.407	34.2	− 35.2
+ 54.33	+ 4.17	121.23
91.337	162.712	

Multiplication

Multiplying decimals is the same as multiplying any other whole number. Once the answer is determined, you insert the decimal according to the total number of positions taken by all the decimals in the original numbers, counting from right to left.

EXAMPLE 12

4.236
+ 2.41
10.20876

There are five decimal places total (three in the first number and, two in the second). Counting from the right, you place the decimal point after the first zero and before the 2.

Division

In order to divide decimals, you must make them whole numbers before dividing them. To do this, you move the decimal point to the right until a whole number is made. If both numbers do not have the same amount of decimal places, you will move the decimal in the number with the largest amount of decimal places first. Next, count the number of places it was moved and then move the decimal in the second number the same number of positions, filling in the “extra” spots with zeros. Now you can perform the division.

■ **EXAMPLE 13A**

$\frac{4.234}{1.67} = 4234 \div 1670 = 2.5353$

The decimal places in the numerator are three, so move it to the right that many places. The denominator has only two, so you will move it two places plus one more by putting a zero in that position.

■ **EXAMPLE 13B**

$\frac{5.426}{3} = 5425 \div 3000 = 1.808$

Rounding

When you are multiplying and dividing decimals, your answer may have more decimal digits than your original numbers. For measurement of liquids, you will generally round to the nearest tenth. Tuberculin and insulin syringes allow you to measure to the nearest hundredth. Solid measurements can be accurately measured to the hundredth.

To perform this calculation, you will look at the number to the immediate right of the one you wish to round. If that number is four or less, it will remain the same. If the number to the right of the one you wish to round is five or higher, you will add one to that number.

■ **EXAMPLE 14**

Round to the nearest tenth.

14.234 → 14.2 The number to the immediate right of the "tenth" position is a 3, so the number in the "tenth" position remains the same (2).

1.5932 → 1.6 The number to the immediate right of the "tenth" position is a 9, so the number in the "tenth" position will increase by one.

When performing multiplication and division of decimals where the result is being used for measuring purposes of liquid medications, then the rounding of the decimal will be determined by the equation itself. If you are multiplying a number with four decimal positions and another one with only two decimal positions, then the final answer will retain only two decimal positions. **An equation's answer is only as accurate as the number with the least decimal positions.**

■ **EXAMPLE 15A**

Multiply the following.

$3.5433 \times 2.42 = 8.574786 = 8.57$

The answer results in a number with six decimal positions. The first number has four decimal positions, and the second number has only two. Therefore, the answer can have only two decimal positions. Following the rules of rounding numbers given previously, the four (4) is used to determine the rounding of the second decimal position (7). Because the number is less than five, it will remain the same.

EXAMPLE 15B

$4.632 \times 8 = 37.056 = 37$

Practice Problems 2.3

Round to the correct decimal position.

1. $23.4 \times 42.87 =$
2. $8.56 \times 39.2 =$
3. $146 \times 15.58 =$
4. $1.4756 \times 2.4 =$
5. $3.4 \times 4.21 \times 1.8 =$
6. $4.56 \div 2.3 =$
7. $45.98 \div 9.135 =$
8. $7.34 \div 4.9 =$
9. $5.3214 \div 3.87 =$
10. $24 \div 4.34 =$

Significant Figures

A significant figure is a digit of known value in a number that can be accurately measured. A number usually consists of one or several significant figures, or digits, plus one more digit that cannot be considered accurate but is needed to hold a position in the number. Consider the following rules when determining significant figures:

1. All leading zeros are not significant.
2. All zeros within (surrounded) by digits are significant.
3. Count all digits from left to right excluding leading zeros.
4. Zeros at the end of a digit may or may not be significant depending on the accuracy of the measuring tool.

EXAMPLE 16

Remember: Leading zeros NEVER count!

Remember: Trailing zeros may or may not count—it depends on the accuracy of the measurement.

0.0125 = 3 significant figures (two leading zeros)

1.032 = 4 significant figures (zero within digits)
18,543 = 5 significant figures
4.34 = 3 significant figures
0.9 = 1 significant figure (leading zero)
3.0 = 1 significant figure

or

2 significant figures—*if* it is stated that the calculation is accurate to the "tenth" position

8.430 = 3 significant figures

Practice Problems 2.4

Determine the significant figures—final zeros are *not* significant.

1. 0.254
2. 1.085
3. 43.2
4. 2.0001
5. 3.0
6. 902
7. 5439.1
8. 1.2040
9. 22,432.2
10. 0.0003
11. 700
12. 0.019040

Round to three significant figures.

13. 57.82
14. 49.074
15. 0.5486
16. 0.00486
17. 1.438
18. 1.899
19. 101.36
20. 42.98

CHAPTER 2 QUIZ

Perform the following calculations. Simplify when possible.

1. $\frac{2}{4}+\frac{2}{3}=$
2. $\frac{4}{5}+\frac{7}{8}=$
3. $\frac{6}{20}+\frac{4}{10}+\frac{4}{5}=$
4. $\frac{9}{14}-\frac{3}{14}=$
5. $2\frac{5}{6}+3\frac{1}{4}=$
6. $\frac{5}{6}\times\frac{3}{8}=$
7. $\frac{10}{75}\times\frac{20}{100}=$
8. $\frac{7}{15}\times\frac{3}{4}=$
9. $8\div\frac{2}{3}=$
10. $3\frac{1}{3}\times4\frac{1}{4}=$
11. $\frac{5}{9}+\frac{2}{9}=$
12. $\frac{2}{15}+\frac{8}{15}=$
13. $\frac{7}{8}-\frac{3}{4}=$
14. $\frac{5}{8}-\frac{1}{3}=$
15. $4\frac{2}{13}+3\frac{7}{26}=$
16. $\frac{11}{14}\times\frac{1}{4}=$
17. $\frac{2}{3}\times\frac{1}{4}\times\frac{7}{8}=$
18. $\frac{5}{8}\div\frac{1}{2}=$
19. $\frac{2}{7}\div4=$
20. $6\frac{1}{4}\div3\frac{1}{3}=$

Perform the following calculations. Round to the proper decimal position.

1. $23.6 + 42.87 =$
2. $346.42 + 2.68 + 49.572 =$
3. $423.73 - 284.6 =$
4. $26.84 - 14.82 =$
5. $3.1 \times 6.82 =$
6. $7.42 \div 3.6 =$
7. $9.8624 \div 4.36 =$
8. $17 \div 2.46 =$
9. $5.62 \times 3.9 =$
10. $8.92 \times 2.6 =$

Determine the significant figures—final zeros are *not* significant.

1. 0.426
2. 3.028
3. 7326.4
4. 1.8604
5. 7.0

Round to three significant figures.

1. 52.083
2. 1.377
3. 0.6491
4. 1.888
5. 234.2