Preface

This text provides a complete set of teaching materials for a course in multivariable calculus that incorporates recent curricular and pedagogical developments in the teaching of calculus. The intended audience is mathematics, science, and engineering majors who have had the equivalent of a full year of single variable calculus but who have not had a course in linear algebra. Typically, these students are in their second or third semester of college. On a curricular level, the driving force behind this text is the desire to bridge the gap between mathematical concepts and their use in real-world applications outside of mathematics. Consequently, the ideas of multivariable calculus are presented in a context that is informed by their nonmathematical applications. Pedagogically, the text incorporates collaborative learning strategies and encourages the sophisticated use of technology.

Each chapter of the text begins with a collaborative learning exercise that is designed to engage students as active participants in the development of their understanding of the material. The collaborative exercise introduces an application of the material in the chapter or a key concept for the chapter. Each is designed to be used collaboratively by students working in groups of two to four. While some of the questions in the exercises require numerical answers, most of the questions involve an analysis of a problem or application that requires a prose answer. Thus an emphasis is placed on the development of the student's ability to communicate mathematically, both orally and in writing.

An additional set of collaborative learning exercises is available online from Jones & Bartlett Learning. Many of these exercises introduce new concepts and are to be used before the students read the corresponding section of the text. There are also several other exercises that are designed primarily to use the graphical and computational capabilities of the software package Maple™ to reinforce and explore topics that have been covered in class or to investigate an extended modeling application. Although the text can be used without the collaborative exercises, it becomes a more effective teaching tool when some collaborative work is introduced. The online instructor's manual contains summaries of the collaborative exercises and suggestions for their use in tandem with the text.

Content, Theory, and Method

The central theme of the text—the connections between mathematics and the sciences—is present throughout. The discussion of a topic is initiated and guided by a consideration of its uses outside of mathematics. As the mathematical ideas are developed, the text returns to their application in the sciences in extended examples and exercises. Furthermore, selected applications appear in the collaborative learning exercises to be used as in-class discussions or as the basis for extended modeling exercises. Applied topics are chosen both from the physical sciences and the life sciences and include traditional applications from mechanics as well as, for example, recently developed applications from physiology. A particular goal of this material is to provide sufficient information for students to appreciate the need to interpret a mathematical answer to a scientific question in its original context and to begin to carry out this interpretation themselves.

An effort has been made to present the material clearly and at a level of sophistication appropriate for the audience. It is important that, by the end of the third semester of calculus, students appreciate the need for precise mathematical statements and the need for justifying or proving these statements. Thus, from the beginning, the text uses set notation and the language and symbolism of functions, and definitions, propositions, and theorems are stated using precise mathematical language. The text also provides proofs of many important results in multivariable calculus, including, for example, the major theorems of line and surface integration, but it does not include concepts or proofs that require analytic capabilities beyond those of third-semester students, for example, the implicit and inverse function theorems. Furthermore, since the intended audience, in general, will not have had any formal experience with linear algebra, the focus is on $\mathbb{R}^2$ and $\mathbb{R}^3$ rather than $\mathbb{R}^n$ and vector spaces. Consequently, the algebra of linear transformations is not employed in the text. For example, differentiability of real-valued functions is phrased in terms of the existence of a linear approximation l to f, rather than the existence of a linear transformation that is called the derivative. Similarly, integration in nonrectangular coordinates is presented via Riemann sums rather than as an application of change of coordinates and the Jacobian of a coordinate transformation.

Vector fields are introduced early in the text, in Chapter 2, and then used in a variety of models, including the Lotka–Volterra predator–prey model and a phase plane model of a simple pendulum. By considering flow lines of a vector field and a graphical classification of the critical points of a vector field, we are able to make maximum use of the gradient vector field in discussing critical points of differentiable functions of two or three variables, the chain rule, and constrained optimization problems. Thus vector fields are familiar objects by the time we move to the study of line and flux integrals.

In addition to incorporating collaborative learning models, the text uses other developments in mathematical pedagogy. In particular, it emphasizes that functions can be represented numerically and graphically (in several ways) rather than only as a symbolic expression. This is particularly important for this text because most scientific applications begin with a collection of data that is represented numerically or graphically or a qualitative analysis that gives rise to differential equations. Indeed, it is a rare problem in the sciences that admits a closed-form symbolic solution. Thus the text contains numerical and graphical examples from outside of mathematics that cannot be expressed symbolically. Of course, some representations are better suited to thinking about particular mathematical ideas than others. For example, contour plots are useful when thinking about rates of change of functions, whereas data sets and density plots are better suited to discussions of total accumulation and Riemann sums.

The extensive end-of-section exercises include standard symbolic manipulations, simple proofs and verifications, problems that require a computer algebra system, and questions that require prose explanations. The exercises range in difficulty from simple calculations to more involved questions that do not follow templates provided in the text. The exercises both reinforce and extend the textual material. They also provide ample opportunities to continue the discussion of the applications of mathematical ideas to the sciences by asking questions about the particular examples that appear in the text, by varying data and models, and by presenting entirely new examples of the same type. Beyond simply asking students to carry out additional calculations, these problems ask students to explain and interpret their results in light of the phenomena being analyzed.

To illustrate the variety of ways that applications are used in the text, we offer the following brief summary of selected examples from Chapter 3, *Differentiation of Real-Valued Functions*. We begin the chapter with a discussion of an altitude function. In the opening collaborative exercise, students are asked to analyze a contour plot of a mountain in New Hampshire. In particular, they are asked to consider points that are local maxima and saddle points and to describe the contours around these points. They are also asked to consider a path along the contour plot and describe the corresponding path along the mountain. In this way, they are introduced to some of the key ideas of the chapter, critical points and rates of change, in a familiar context.

In Section 3.1, where real-valued functions are encountered formally for the first time, the first example focuses on how the language of functions might be used to describe quantities that depend on latitude and longitude, to describe physical laws, to describe quantities that depend on time and one or more other quantities, or to describe surfaces in $\mathbb{R}^3$. Subsequent examples in the same section explore these and other applications in more detail and connect basic mathematical questions we might ask about functions to questions that arise in the scientific context. Thus Example 3.5 discusses the motion of a vibrating string and, in particular, how we might describe traveling waves and standing waves with functions of time and position, and Example 3.6 examines implicitly defined surfaces by considering the isobars of sound pressure of an acoustic monopole. Later in the chapter, as the various notions of differentiation are introduced, they are used to carry out more sophisticated analyses of the applied situations. For instance, Example 3.11

computes the directional derivatives of the rms pressure of a monopole acoustic source; Example 3.13 gives a physical interpretation of the partial derivatives of the model of the vibrating string; Example 3.20 considers the question of continuity for the potential energy function of a system consisting of two molecules; and Example 3.22 considers the question of differentiability for a model of the intracellular concentration of calcium. Beyond simple questions of finding extreme values of physical quantities, the examples and accompanying exercises explore deeper questions, including, for example, why we might want the potential energy function of a molecular system to be discontinuous, or what possible physiological explanations there are for intracellular calcium concentrations to be nondifferentiable in time.

It may, of course, be necessary to employ a range of mathematical concepts and techniques to develop a complete understanding of a physical system. In order to emphasize this point, many physical systems appear in more than one chapter. For example, the acoustic systems that are introduced in Chapter 3 appear again in Chapter 7 when we consider absorption, reverberation, and total acoustic power. Systems of charged particles are first encountered in Chapter 1, *Euclidean Space and Vectors*, in the construction of the vector representing the force on a charged particle. They appear again Chapter 3, *Differentiation of Real-Valued Functions*, where we are concerned with the potential energy function of a system of two charged particles; in Chapter 4, *Critical Points and Optimization*, where we consider the extreme values of the potential energy of a system subject to a constraint; in Chapter 5, *Integration*, where we compute the potential energy of a planar charge distribution; and in Chapter 6, *Integration on Curves*, and Chapter 7, *Integration on Surfaces*, where we consider conservative forces and compute the total flux of an electric field in the plane and in space.

The Instructor's Materials

- The online instructor's manual contains commentaries on each chapter and each collaborative exercise and can be found at http://go.jblearning.com/damiano. The commentaries summarize the content of each chapter and suggest possible uses of the collaborative exercises in the classroom, in the laboratory, or outside of class in extended assignments. In addition, there is a daily schedule for each chapter that suggests how to coordinate textual material and collaborative exercises in order to take full advantage of the design of the project. This includes days for lectures on material that is most effectively presented in a lecture. It indicates what the students should have seen in the text and in class, either in discussion or lecture, prior to using a particular discussion. It also gives a summary of material that should be included in an introduction to the discussion and suggestions for summarizing or for follow-ups of the discussion.
- Solutions to the text's exercises are available for qualified instructors.
- PowerPoint® lecture slides are organized by chapter.
- WebAssign™, developed by instructors for instructors, is a premier independent online teaching and learning environment, guiding several million students through

their academic careers since 1997. With WebAssign, instructors can create and distribute algorithmic assignments using questions specific to this textbook. Instructors can also grade, record, and analyze student responses and performance instantly; offer more practice exercises, quizzes, and homework; and upload additional resources to share and communicate with their students seamlessly such as the PowerPoint slides supplied by Jones & Bartlett Learning. For more detailed information, please visit www.webassign.net.

- As an added convenience this complete textbook is now available in eBook format for purchase by the student through WebAssign.

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