# ONLINE ERRATA AND ADDENDA
for
# ELEMENTS OF REAL ANALYSIS
(First Printing)

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Subsequent to the first printing of this book in April 2010, I discovered some errors and a few places requiring added clarification. Most of these occur in the "Answers and Hints for Selected Exercises," in Appendix C. While most are relatively minor, a few are serious enough to merit calling them to your attention. By means of this *Online Errata and Addenda*, I will keep you posted on the latest complete listing of all the errors of which I am aware, and their corrections. I will update the list from time to time, as new errata and addenda are needed. Please note that

(a) Most of these corrrections have been incorporated into the **second printing**.

(b) Pages 448 and 449 do not require the changes suggested in earlier versions of this errata.

If you discover additional errors, please call them to my attention at cgdenlinger@hotmail.com.

# ERRATA AND ADDENDA

## Chapter 1 - The Real Number System

**Page 16.** In the statement of Theorem 1.2.13, the first (d) should be (c).

**Page 33.** In the statement Theorem 1.5.9, remove the word "Archimedean.". In the paragraph immediately above this theorem, remove the words "continues in the vein of Throrem 1.5.2. It"

**Page 43.** Exercise 4: Replace "$\sup A$" by "$\inf A$".

**Page 55.** Sixth line from the bottom: replace " $< \frac{3}{n-3}$ " by " $= \frac{3}{n-3}$ ".

## Chapter 2 - Sequences

**Page 70.** Exercise 8: Replace entire exercise by "Suppose $\{a_n\}$ is bounded and $\{b_n\}$ converges. Prove that $\{a_n b_n\}$ must be bounded, but find an example to show that it need not converge."

**Page 78.** Remove the asterisk on Theorem 2.3.10. This result is used enough times later in the book to warrant removing the asterisk. However, you may wish to keep the asterisk on the proof.

**Page 82.** Replace bracket symbols [ ] (for the greatest integer function) by the more current symbols $\lfloor\ \rfloor$.

**Page 132.** Line 4: Footnote marker 18 refers to the note on the bottom of the previous page.

## Chapter 3 - Topology of the Real Number System

**Page 174.** Exercise 12: insert the word "if" between "that" and "$A$".

Exercise 16 should read: Show that for measurable $A$, the ($\Rightarrow$) direction of ($\mu 9$) follows from ($\mu 1$) – ($\mu 8$).

**Page 175.** Renumber Exercise 20 as Exercise 22, and add two new exercises:

Exercise 20. Prove that a set $A$ of real numbers is nowhere dense iff every nonempty open interval has a nonempty open subinterval disjoint from $A$.

Exercise 21. Prove that the union of two nowhere dense sets is nowhere dense.

## Chapter 4 - Limits of Functions

**Page 193.** Line 13: Change "Exercise 8" to "Exercise 6".

**Page 202.** Exercise 15, line 1: replace "$f$ and $g$" by "$f$, $g$, and $f/g$".

**Page 215.** Exercise 4: replace "Exercises 8-20" by "Exercises 5 and 6".

## Chapter 5 - Continuous Functions

**Page 256.** Exercise 27 can be strengthened as follows: Change "has the extreme value property" to "is bounded".

**Page 265.** Exercise 11 (d): Change "$x \sin x$" to "$x \sin\left(\frac{1}{x}\right)$".

**Page 274.** Exercise 10: Change "14" to "16" in two places.

**Page 292.** Definition 5.7.4: After "$\sup f(A) - \inf f(A)$" insert "$= \sup\{f(x\} - f(y) : x, y \in A\}$".

Exercise 5.7.8: After "$\mathbb{R} \cup \{+\infty\}$" insert "and equals $\inf\{\omega_{f,x_0}(\varepsilon) : \varepsilon > 0\}$".

Theorem 5.7.11: Insert "If $\mathcal{D}(f)$ is closed," at the beginning.

In (a) of the proof, remove "[See Exercise 5.1.23.]"

## Chapter 6 - Differentiable Functions

**Page 316.** In Exercise 20 (d), the hypothesis "$\lim_{x \to b^-} f'(x) = \lim_{x \to a^+} f'(x)$" can be replaced by the weaker hypothesis "$\lim_{x \to b^-} f'(x)$ exists."

**Page 337.** Theorem 6.5.14: In line 4, after the words "*That is,*" insert "$\forall x \in I$,".

**Page 339.** In Exercise 6, lines 3 and 4, replace "$T_5(x)$," "$R_5(x)$," "$T_7(x)$," and "$R_7(x)$" by "$T_4(x)$," "$R_4(x)$," "$T_6(x)$," and "$R_6(x)$", respectively.

## Chapter 7 - The Riemann Integral

**Page 358.** Theorem 7.1.4: At the end of line 1, add "and $A \ominus B = \{a - b : a \in A, b \in B\}$."

Add "(c) If $A$ is bounded above and $B$ is bounded below, then $\sup(A \ominus B) = \sup A - \inf B$."

**Page 380.** Proof of Theorem 7.3.11: in line 7, replace "$\mathcal{Q}$" by "$\mathcal{Q}_m$".

**Page 387.** Theorem 7.4.2: in the last line of the proof, replace $\overline{\int_c^b} f$ by $\overline{\int_a^b} f$.

**Page 389.** In line 5 of (b), replace $\int_a^b$ by $\int_a^b f$.

**Page 401.** Exercise 1 (d): Replace "Theorem 7.5.1 (c)" by "Theorem 7.5.1 (b)".

**Page 407.** In line 14, replace "Theorem 7.5.5" by "Corollary 7.5.5".
In line 15 replace "Corollary 7.5.2" by "Theorem 7.5.2 (b)".

**Page 421.** Exercise 20, line 6: Replace "$f$ is differentiable ..." by "$F$ is differentiable ...".

**Page 434.** Definition 7.8.1: in line 3, replace $\int_c^b$ by $\int_c^b f$.

**Page 435.** In line 1, replace $\lim_{c \to b^-} \int_a^c$ by $\lim_{c \to b^-} \int_a^c f$.

**Page 436.** Replace Example 7.8.7 by: $\displaystyle\int_0^1 \frac{1}{x^2 + \sqrt{x}}\, dx$ converges because $\forall x \in (0, 1]$, $0 \le \dfrac{1}{x^2 + \sqrt{x}} \le \dfrac{1}{\sqrt{x}}$ and $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\, dx$ was shown to converge in Example 7.8.4.

**Page 437.** Exercise 3 (b): Replace $\sqrt{x}$ by $\sqrt[3]{x}$.

**Page 440.** Theorem 7.8.16: Replace "(where $a = \infty$ or $b = -\infty$)" by "(where $a = -\infty$ or $b = +\infty$)".

**Page 445.** Remarks 7.9.3: Replace "Let $a > 0$" by "Suppose $f : \mathcal{D}(f) \to \mathbb{R}$ is bounded".

Part (c): Replace "$[a, b]$" by "$\mathcal{D}(f)$".

Part (e): Begin with "If $\mathcal{D}(f)$ is closed, then".

**Page 448.** Fourth paragraph, line 2: Insert "finite" between "a" and "collection".

**Page 452.** Exercise 7, line 3: Replace "$\chi_n$" by "$\chi_{I_n}$".

## Chapter 8 - Infinite Series of Real Numbers

**Page 465.** Theorem 8.2.10 (b): Replace "all but finitely many $n$" by "infinitely many $n$".

**Page 466.** Lines 5 - 7: Replace the entire part (b) of the proof by the following:

Suppose $\frac{a_{n+1}}{a_n} \geq 1$ for infinitely many $n$. Then $\{a_n\}$ has a subsequence $\{a_{n_k}\}$ such that $\forall k$, $\frac{a_{n_k+1}}{a_{n_k}} \geq 1$; i.e., $a_{n_{k+1}} \geq a_{n_k}$. Then $\{a_{n_k}\}$ is a monotone increasing sequence of positive terms, so $a_{n_k} \not\to 0$. But then $a_n \not\to 0$. By the general term test, $\sum a_n$ diverges.

**Page 477.** In the seventh line from the bottom replace " $+ a_{2n}$" by " $- a_{2n}$".

**Page 535.** Exercise 18, line 2: Replace "$|i|$, $|j|$" by "$|r|$, $|s|$".

## Chapter 9 - Sequences and Series of Functions

**Page 579.** Line 8: remove the subscript from $\|f\|_1$; it should be just $\|f\|$.

## Appendix B - Sets and Functions

**Page 618.** Exercise 7, line 2: $\bigcup_{\lambda\in\Lambda} A_\lambda^c$ appears twice. Chance the second occurrence to $\bigcap_{\lambda\in\Lambda} A_\lambda^c$.

## Appendix C - Answers and Hints for Selected Exercises

NOTE: Fully corrected solutions are provided in the (online) Instructors' Solutions Manual, which is updated regularly.

EXERCISE SET 1.6-B

**Page 640**, Exercise 1 (b). Replace "$\cdots$ so $-v \leq u$. That is, $v \geq -u$." by "$\cdots$ so $-v \geq u$. That is, $v \leq -u$."

EXERCISE SET 2.1

**Page 640**, Exercise 1 (a). Replace "$\frac{1}{81}$" by "$\frac{1}{64}$".
Exercise 2 (c). Replace "700" by "701" and "$14,000$" by "$14,001$".

EXERCISE SET 2.7

**Page 646**, Exercise 3. In line 2, replace "$C^n$" by "$C^{n-1}$" and "$C^{n-m}$" by "$C^{n-m-1}$".

EXERCISE SET 2.8

**Page 647**, Exercise 9. In line 1, replace the word "infinite" by the word "uncountable".

EXERCISE SET 2.9

**Page 648**, Exercise 1 (g). Should be "$+\infty, 0$".

EXERCISE SET 3.1

**Page 648**, Exercise 6 (l). Replace "$A^b = \mathbb{Q} \cap [0,1]$" by "$A^b = [0,1]$".

EXERCISE SET 3.2

**Page 650**, Exercise 16 (l). Replace "$\mathbb{Q} \cap [0,1]$" by "$[0,1]$".

EXERCISE SET 3.3

**Page 651**, Exercise 7, line 2. Insert "possible" between "only" and "cluster".

EXERCISE SET 4.1

**Page 653**, Exercise 2, line 1. Should read "Choose $\delta$ as follows: . . ."

EXERCISE SET 4.2

**Page 653**, Exercise 1, lines 1 and 2: Change "$|x-0|$" to "$|x-x_0|$".

**Page 654**, Exercise 19, lines 1, 2 and 3: Change "$|x-0|$" to "$|x-x_0|$" (three places).
Exercise 21, line 3: Change "$|x-0|$" to "$|x-x_0|$".

EXERCISE SET 4.4-A

**Page 655**, Exercise 5 (b): Change "$(x_0,+\infty)$" to "$(-\infty,x_0)$".

EXERCISE SET 4.4-B

**Page 656**, Exercise 4, line 3: Change "$2m/3$" to "$2M/3$".

**Page 656**, Exercise 9 (a), line 2: Change "$\lim_{x\to-\infty}$" to "$\lim_{x\to+\infty}$".

EXERCISE SET 5.1

**Page 657**, Exercise 1 (a), line 2: Change "$f(x_0$" to "$f(x_0)$".

**Page 658**, Exercise 17, line 4: Remove "$\therefore$".

EXERCISE SET 5.3

**Page 662**, Exercise 21, line 12: Change "$f$ is continuous on $[a,b]$" to "$f:[a,b]\to\mathbb{R}$ is continuous".

Exercise 27: With the strengthening of the exercise (see entry for Page 256) the proof now simplifies as follows:

($\Leftarrow$) Suppose every continuous $f:A\to\mathbb{R}$ is bounded. Since the function $i:A\to\mathbb{R}$ given by $i(x)=x$ is continuous, $A$ must be bounded. We must prove $A$ closed. Let $x_0\in A^c$. Define $f(x)=\frac{1}{|x-x_0|}$. Then $f$ is continuous on $A$ since $x_0\notin A$, so by hypothesis, $f$ is bounded on $A$. That is, $\exists M>0\ni\forall x\in A$, $f(x)\le M$; i.e., $\frac{1}{|x-x_0|}\le M$. Then $\forall x\in A$, $|x-x_0|\ge\frac{1}{M}$. Thus, $N_{1/M}(x_0)$ contains no point of $A$, so $N_{1/M}(x_0)\subseteq A^c$. $\therefore A^c$ is open, so $A$ is closed.

EXERCISE SET 5.4

**Page 663**, Exercise 3, line 3: Change "$fx_0$)" to "$f(x_0)$".

EXERCISE SET 5.6

**Page 665**, Exercise 9, line 3: After "$+\infty$" add "and $\lim_{x\to0^+}g(x)=0$".
line 4: Change "$+0$" to "$0$" and add "and $\lim_{x\to0^+}f(x)=+\infty$".
Exercise 13, (e): Change "decreasing" to "increasing".

EXERCISE SET 6.1

**Page 667**, Exercise 13, line 2: Replace "$\lim_{x\to0^-}\frac{1}{x}$" by "$\lim_{x\to0^-}\frac{-1}{x}$".

EXERCISE SET 6.2

**Page 668**, Exercise 19, line 5: Replace "$f'(u)$" by "$f'(u_0)$".

EXERCISE SET 6.3

**Page 669**, Exercise 5:

(a) Replace $(-2, \frac{1}{2})$, $(3, +\infty)$, $(-\infty, -2)$ and $(\frac{1}{2}, 3)$ by $[-2, \frac{1}{2}]$, $[3, +\infty)$, $(-\infty, -2]$ and $[\frac{1}{2}, 3]$.

(c) Replace $(-1, 0)$ and $(0, 1)$ by $(-1, 0]$ and $[0, 1)$.

(e) Replace $(-\infty, -0)$ and $(0, \infty)$ by $(-\infty, 0]$ and $[0, \infty)$.

(f) Replace $(-1, 1)$, $(-\infty, -1)$ and $(1, \infty)$ by $[-1, 1]$, $(-\infty, -1]$ and $[1, \infty)$.

Exercise 7 (a), line 3: Replace "$\lim_{h\to 0}$" by "$\lim_{x\to 0}$" (two places).

EXERCISE SET 6.4

**Page 670**, Exercise 27, line 8: Replace "$c, d > 0$" by "$c, d \in I$".

EXERCISE SET 6.5

**Page 671**, Exercise 9, line 1: Replace "$|x| < 2$" by "$|x| \leq 2$".

line 3: Remove " $\frac{e^2}{7!}2^7 =$ " (but do not remove it from line 2).

Exercise 11 (a): Replace all three occurrences of "0.005" by "0.0005".

**Page 672**, Exercise 11 (b): Replace all three occurrences of "0.005" by "0.0005".

In the table, replace "4" by "6", and replace "5" by "7".

In the table, replace ".023" by ".00054", and replace ".0038" by ".000067".

In the last line, replace "$n \geq 5$" by "$n \geq 7$", and replace "$T_5(x)$" by "$T_7(x)$".

Exercise 15, line 3: Replace "$y =$" by "$y_n =$".

EXERCISE SET 6.6

**Page 674**, Exercise 15, part (b): Replace all five occurrences of " $\lim_{x\to x_0^+}$ " by " $\lim_{x\to 0^+}$ ".

Part (b), (ii): In line 2, replace "Now $\frac{1}{x}p_k\left(\frac{1}{x}\right)$ is $\cdots$ " by "Now $p_k\left(\frac{1}{x}\right)$ is $\cdots$ ".

EXERCISE SET 7.2

**Page 675**, Exercise 5: Replace "$a$" by "2", and "$b$" by "9" (5 times).

In line 2, replace "$\varepsilon > 0$" by "$0 < \varepsilon < 5$".

Exercise 7 (b), line 2: Replace "$\frac{438}{64}$" by "$\frac{219}{32} \approx 6.85$", and replace "$\frac{294}{64}$" by "$\frac{339}{32} \approx 10.59$".

Exercise 7 (c): In line 1 replace "$\frac{79}{9}$" by "$\frac{70}{9}$", and "$\frac{61}{9}$" by "$\frac{58}{9} = 6.\overline{4}$".

In line 2 replace "$\frac{158}{9}$" by "$\frac{140}{9}$", and "$\frac{316}{27}$" by "$\frac{298}{27} = 11.\overline{037}$".

**Page 677**, Exercise 13: Replace the entire solution by " For $f, g$ satisfying the hypotheses, and all partitions $\mathcal{P}$ of $[a, b]$, $\underline{S}(f, \mathcal{P}) \leq \underline{S}(g, \mathcal{P})$ and $\overline{S}(f, \mathcal{P}) \leq \overline{S}(g, \mathcal{P})$, so $\underline{\int_a^b} f \leq \underline{\int_a^b} g$ and $\overline{\int_a^b} f \leq \overline{\int_a^b} g$".

EXERCISE SET 7.3

**Page 678**, Exercise 3, line 2: Replace "infimum" by "supremum".

Exercise 9, line 1: Replace "$[a, b]$" by "$[0, 1]$".

**Page 679**, Exercise 10, line 2: Replace "$i$" by "$k$" (3 places).

Exercise 15: In line 1 replace "line 7" by "line 9".

Exercise 17: In line 5 replace "$k \geq \max\{m, n\}$" by "$k = mn$".

EXERCISE SET 7.4

**Page 681**, Exercise 15: Replace the first occurrence of "$[0,1]$" by "$(0,1)$".

EXERCISE SET 7.5

**Page 682**, Exercise 3, line 6: Replace "$[0,a]$" by "$[-a,a]$".

EXERCISE SET 7.6

**Page 684**, Exercise 17, right end of line 4: Replace "$f(c)(c-$" by "$f(a)(c-$".

Exercise 23, right end of line 2: Replace "$(-1)^{2n-1}p\psi^{(2n+1)}(x)$" by "$(-1)^{2n}p^0\psi^{(2n+1)}(x)$".

EXERCISE SET 7.8-A

**Page 685**, Exercise 1, line 1: After "$a<c<b$, insert "but not on $[a,b]$."

Exercise 3 (b): Replace "2" by "3/2".

(f): Replace "$2\sqrt{8}$" by "$\sqrt{8}$".

(i): Replace "diverges to $+\infty$" by "diverges: $\int_1^2 f$ diverges to $+\infty$; $\int_{-2}^1 f$ diverges to $-\infty$."

(k): Replace "to $2(e-1)$" by "by comparison with $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$, which converges to $2(e-1)$."

EXERCISE SET 7.8-B

**Page 685**, Exercise 1 (h): Replace "$1/e$" by "$2/e$".

(j): Replace "$2-1/e$" by "2".

Exercise 5 (b): To the left of the final integral on the line, replace "$-1$" by "$-\frac{1}{\pi}$".

(c): Replace "$[\pi,(n+1)\pi\}$" by "$[\pi,(n+1)\pi]$".

(d): Replace "$[n\pi,(n+1)\pi]$" by "$[n\pi+\frac{\pi}{6},(n+1)\pi-\frac{\pi}{6}]$" in both lines.

(e): (page 686, line 2): Insert "$\frac{1}{2}$" before "$\sum_{k=1}^{n}\frac{1}{k+1}$".

EXERCISE SET 7.9

**Page 686**, Exercise 1, line 5: Replace "$I_{(n+1)/2}$ if $n$ odd" by "$J_{(n+1)/2}$ if $n$ odd".

Exercise 3: Replace entire solution by "Apply Exercise 2 to $|f-g|$."

**Page 687**, Exercise 6, line 4: Replace "rational number" by "member of $A$".

EXERCISE SET 8.1

**Page 687**, Exercise 11: Replace "$\frac{1}{2}+\frac{1}{3}$" by "$1+\frac{1}{2}$" and replace "$\frac{5}{12}$" by "$\frac{3}{4}$".

EXERCISE SET 8.2

**Page 688**, Exercise 33 (a), line 1: Replace "$\ln x > p$" by "$\ln x > 1/p$".

**Page 689**, Exercise 37 (b), line 3: Replace "all but finitely many" by "infinitely many".

EXERCISE SET 8.3

**Page 690**, Exercise 5, line 4: Replace "$\sum |a_n|$" by "$\sum |a_k|$".

Exercise 14 (c), line 1: Replace "$\dfrac{8n^3-8n^2-2n-1}{8n^3+12n^2-2n-3}$" by "$\dfrac{8n^3-8n}{8n^3+12n^2-2n-3}$".

EXERCISE SET 8.4

**Page 692**, Exercise 9 (c), line 2: Replace "$(-1)^{k+1}$" by "$(-1)^{i+1}$", and replace "$(-1)^{k+1}$" by "$(-1)^{k+2-i}$".

line 3: Replace "$\dfrac{1}{1^{3/2}k^{1/2}} = \frac{1}{\sqrt{k}} \geq \frac{1}{k}$" by "$\sum\limits_{i=1}^{k} \dfrac{1}{k^{3/2}k^{1/2}} = \frac{k}{k^2} = \frac{1}{k}$, and $\sum\limits_{i=1}^{k} \frac{1}{k}$ diverges."

EXERCISE SET 8.6

**Page 694**, Exercise 17 (b): Replace the entire answer by
"Converges absolutely in every interval $(2n\pi, (2n+1)\pi)$, $n \in \mathbb{Z}$. It converges when $x = (2n+1)\pi$ but diverges when $x = 2n\pi$."

EXERCISE SET 8.7

**Page 694**, Exercise 1, line 2: Replace "$\int_x^c$" by "$\int_c^x$".

**Page 695**, Exercise 9, line 2: Replace "$(x-c)^{2k+4}$" by "$(x-c)^{2k+1}$".

EXERCISE SET 9.1

**Page 696**, Exercise 5, line 1: Replace "$f(x) = |x|$ if $|x| \geq 1/n$" by "$f(x) = |x| - 1/n$ if $|x| \geq 1/n$".

EXERCISE SET 9.2

**Page 696**, Exercise 7, line 2: Replace "$fn$" by "$f_n$".

**Page 697**, Exercise 9 (b), line 1: Replace "$\in \mathbb{R}$" by "$> 0$".

line 2: Replace "on $\mathbb{R}$" by "on $(0, +\infty)$".

(g): Replace the entire entry with the same answer as (h).

(i): Insert the word "closed" between "any" and "interval".

**Page 698**, Exercise 17, Replace every occurrence of "$<$" by "$\leq$" (16 places).

lines 1 & 4: Replace "$(-1)^k$" by "$(-1)^{k+1}$" (3 places).

line 5: Replace "$\sum\limits_{k=1}^{n+1}(-1)^{k+1}x^{k+1}$" by "$\sum\limits_{k=1}^{n+1}(-1)^{k+1}x^k$."

EXERCISE SET 9.3

**Page 699**, Exercise 9, lines 1 & 2: Replace "$\sum\limits_{k=1}^{n}$" by "$\sum\limits_{k=1}^{\infty}$" (3 places).

Exercise 15 (b): Replace "$\sum\limits_{k=1}^{n}$" by "$\sum\limits_{k=1}^{\infty}$" (5 places).

**Page 700**, Exercise 15 (c): Replace "0" by "1" (2 places).

APPENDIX

EXERCISE SET A.1-A

**Page 700**, Exercise 15, line 2: In (d) replace "$(E \Rightarrow W)$" by "$(A \Rightarrow W)$".

EXERCISE SET A.1-B

**Page 701**, Exercise 13: The answer given is the answer to Exercise 14. The answer to Exercise 13 is

| $P$ | $Q$ | $P \vee Q$ | $\sim Q$ | $\sim Q \Rightarrow P$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | T | T | T |
| F | T | T | F | T |
| F | F | F | T | F |

(3rd and 5th columns identical)

EXERCISE SET B.1

**Page 705**, Exercise 7 (e), line 2: Replace "$(-\infty, -1]$" by "$(-\infty, 1]$".

EXERCISE SET B.2

**Page 706**, Exercise 5, line 1: In (c) replace "$(-4, -2]$" by "$(-4, -2] \cup \{0\}$".
line 2: In (d) replace "16" by "$f^{-1}(4)$",
and replace "$(-\infty, 0] \cup \big(f^{-1}(2), f^{-1}(4)\big)$" by "$(-\infty, f^{-1}(4))$".

# BIBLIOGRAPHY

**Page 715**, [99] Replace "*Orthogonal Functions*" by "*Orthogonal Expansions*".

# INDEX

**Page 729**, denseness of one set in another: add page 580.
**Page 730**, first category set: add page 580.
**Page 731**, greatest integer: add page 82.
greatest integer function: add page 82.
**Page 733**, nowhere dense set: add page 580.
**Page 735**, second category set: add page 580.
**Page 736**, dense: add page 580.
first category: add page 580.
second category: add page 580.