Preface

A purpose of *Mathematical Modeling for the Scientific Method* is to clarify the connection between deductive and inductive reasoning as used in mathematics and science. Critical ideas, when first introduced, will be *italicized*, however many definitions will be implied before being formally stated. Students should inquire with the instructor to clarify the meaning of anything new, or concepts insufficiently explained in the text. The goal is to be introductory while covering a broad range of techniques and applications. Ideally, one should strive for the least number of steps required before a cognitive concluding statement can be made. However, this is an art, and often it is better to give more information than be too brief.

None of the data presented here should be considered rigourously obtained or confirmed by experiment. Hopefully the reader will be motivated to further explore issues that have been raised. Calculations should require only elementary operations available on typical calculators. The choice of presentation was made for brevity and clarity, allowing a class of students with a variety of backgrounds to experience the material in a manner that unites perspective by the end of the textbook.

This textbook is appropriate for the following courses:

Mathematical Modeling

Calculus for the Biological Sciences

Mathematics for the Chemical and Physical Sciences

Concise Differential Equations/Linear Algebra with Applications

Probability Functions

Statistical Methods in the Sciences

Reference Manual for basic Algebra, Statistics, Linear Algebra, Calculus, Vector Calculus, Differential Equations, and Modeling in the Sciences

Iteration Methods as applied to Science

Mathematical Modeling for the Scientific Method can be used in several settings:

Part I: The textbook begins with several basic and fundamental mathematical spaces. Manipulations using algebra are presented, along with their conclusions. The distinction between variable and parameter is emphasized. Chapter 2 begins the discussion of specific special functions, with an emphasis on those used in the study of probability. Only single-variable functions are considered. The first part then ends with an introduction to statistics, which technically requires only basic algebra, but philosophically explores the issue of confidence and likelihood. Parameters are determined from data, and the concept of a best model can be introduced.

Part II: Chapter 4 continues the discussion of linear equations. When there is more than one variable and one equation to be studied, the equations become systems. Parameters are estimated using matrix equations. This requires the concept of a vector space. In Chapter 5, the calculus of continuous functions is developed. The idea that many functions behave linearly near a point of interest is explored. This allows the use of algebraic techniques to study continuous functions. Families of linearly-independent functions lead to vector spaces. This part ends with the merging of the previous two topics into vector calculus.

Part III: Difference and differential equations are discussed almost interchangeably. In particular, Chapter 7 presents methods for solving differential equations, but also for finding approximations using iteration methods. There is an example of how to modify equations to gain stability. Then the equations that model gravity and electromagnetism are presented. Finally, a series of topics from quarks through population models to galaxies are briefly described, with problems given that are intended to inspire discussion and further exploration.

Mathematical Modeling: The textbook is designed to cover the essential topics that every mathematics teacher should be familiar with. Chapter 1 discusses mathematics spaces, variables, parameters, relations, and transformations, with basic results and simple proofs. Chapter 2 reviews the main functions of science with particular reference on the probability functions. Statistics is introduced, in a minimally rigorous manner, in Chapter 3. Linear and vector algebra is introduced in Chapter 4. This ends the topics that merely require algebra. Chapters 5 and 6 require notions of limits and continuity, although an algebraic approach is used to introduce the concept of linearization. Chapter 7 reviews techniques for solving differential equations. A method of analysis that may find use in equation-modification is presented for the purpose of system stabilization. Also included is a list of elements needed to obtain an iterated solution rigorously.

Calculus for the Biological, Chemical, and Physical Sciences: The latter Chapters 5, 7, and 8 form the foundation for an applications-oriented mathematical modeling course with options to emphasize biology, physics, or chemistry. Sections 8.5–8.11 emphasize biological applications. Sections 8.1–8.2, 8.4, 8.13–8.15 are oriented to physics applications. Section 8.3 emphasizes applications to chemical reactions. Specifically, Chapter 5 can be used as the basis for a calculus for the biological sciences offering, which can be supplemented with subsections of 8.9.

Probability and Statistical Methods in the Sciences: Chapters 2 and 3 support a course in statistical methods for the sciences.

Reference Manual for basic Algebra, Statistics, Linear Algebra, Calculus, Vector Calculus, Differential Equations, and Modeling in the Sciences and Iteration Methods as applied to Science: Chapters 1, 2, and with selections from Chapter 3 form the basis for a general introductory course in mathematics for the sciences. Chapters 4 and 7 form the basis for a concise combined course in differential equations and linear algebra.

The book, as a whole, provides a broad spectrum mathematical reference for quantitative analysis in the sciences.

Anxiety with Mathematics

Mathematics, more than any other field of study, provides a forum for creativity and discovery. However, some of the difficulty with learning topics in math is that every new lesson contains new definitions, properties, rules, applications, etc. It is the freedom and unboundedness of the subject that can often be the most overwhelming for students. We try to mediate this problem by beginning every new topic with an application. In this way, abstraction is arrived at naturally. Indeed, the origins of algebra begin with the idea that one tree and another tree indicates the presence of two trees. This holds for apples or sheep or stars. Thus,

$$x + x \;=\; 2x\ , \quad x = \text{anything.}$$

If x is the number of cartons of eggs, and a farmer has 360 eggs ready for market, then the number of cartons that can be delivered is a solution to the equation

$$12x \;=\; 360\ , \quad x = \text{number of cartons.} \tag{P.1}$$

The formulation of an algebraic equation like this is often more connected to our individual goals than is the actual process of solving for x. Keeping an eye on our true goals, we may be more interested in a qualitative estimate, like

$$12 > 10 \text{ and } x \geq 0,$$

which are obviously true statements for this situation. These observations logically imply that

$$12x \geq 10x.$$

Now, from (P.1) we obtain

$$360 \;=\; 12x \;\geq\; 10x \quad \text{or just} \quad 360 \;\geq\; 10x,$$

which implies $36 \geq x$. Thus, the farmer cannot deliver more than 36 cartons, and disappointment is now justified if the goal was to deliver 40 cartons. The abstractly introduced variable x allows for a series of mechanically logical steps to be performed, at least by the initiated. These types of quick estimates are as much a part of mathematical thought as is the obtaining of the precise $x = 30$ answer. Hence, people think mathematically all the time. However, formalism is typically chosen by history and preference of the original developers in the different subareas. Consequently, mathematical ideas may appear random and imposed. Not only should students not feel excluded from the practice of symbolism, but they need to be allowed to take ownership in the creative process, in particular with the naming of quantities. This is how students can learn to be comfortable working in the abstract.

The goal of this textbook is to give a sense of scope to those areas in mathematics that have found application in science. The trade off is that many definitions will be inferred rather than presented explicitly and deeper results may just be stated, with only references in place of proofs. However, some techniques of proof will be developed so the reader will have an opportunity to obtain a sense of the foundations of mathematics. An objective of scientists, in particular theoretical physicists, is to state a set of *postulates* that then act as *axioms* on which *deductive reasoning* can be performed.[1] The postulates arise from many experiments, which upon reflection suggest certain laws of nature. Once these laws are *axiomized*, the philosopher can explore the consequences. This led Maxwell to create the first unified field theory of electro-magnetism, and from it estimate the speed of light [2]. It also led Dirac to suggest the existence of *anti-particles* many years before their experimental discovery [7]. For the most part, however, scientists look to confirm the postulates, rather than motivate new axioms to explain new phenomena. Thus the logical framework that mathematics provides sets up a *paradigm* of thought into which researchers get locked. This can be a blessing or a curse, a magnifying glass or blinders, liberation or confinement. This textbook will emphasize the notion of mathematical spaces, within which logical conclusions can be rigorously deduced. For the purpose of this epistle, these spaces should provide a mental rock on which to peer across the landscape of experiential truths.

[1] Newton postulated 3 laws of nature [52], thermodynamics is based on 4 laws [39], and quantum mechanics has 6 postulates on the evolution and observation of quantum states [9].

Ethics in Science

One motivation for writing this book was to review topics in mathematics that commonly appear in science. The intent is to reveal a connection between apparently disparate topics in a way that is natural. To some extent, all computations can be reduced to various algebraic actions on variables. Hence, many problems can be formulated in a way that can be resolved using computer programs. This has led many to think that the need for mathematical training has faded away. This text was written with the belief that this will never be the case.

It should be clear that a rationalization of our experiences requires some intellectual structure. The subject of algebra forces the performer of computations to introduce unknown variables to assist in dealing with the abstract. This opens up a world of creativity if only because the selection of y, rather than x, for an unknown variable, is in a sense a personal choice. The significance is that one is faced with the ultimate question: "What do we know?" The only answer, according to Descartes, is that something must exist, even if our experiences are only illusions. Who knows something exists? You do! Thus you exist. Beyond that you can be certain of nothing.

Science intrinsically must deal with uncertainty. This conflicts with the goals of mathematics, which consist of obtaining pure knowledge via the rules of logic. Thus the development and understanding of mathematics requires deduction, whereas the natural sciences continue to expand according to the process of induction. It is statistics that works to bridge the two areas of intellectual inquiry. This is why we have decided to introduce a discussion of this subject early in the text.

Human actions are considered by psychologists to derive from two sources: intent and opportunity. The latter comes from our environment, and is most often out of our control. Intent can have two sources: social obligations and personal attitudes [64]. What others think are our obligations is part of the social norms of our society [74], and again this can be considered, to a large extent, out of our control. However, our attitudes are something we can work on all our lives. Attitudes come from measures that we call values. Good values are known as virtues, and bad values are vices. Assuming that we are driven toward positive values, then how we assign value is intimately connected to our intentions. Even if the ego is subordinate to social norms, our values, good or bad, will play out in actions that may be hard to explain to others, and ourselves. So there is always a need to review and refresh our values on a regular basis. It is important to continually develop our personal opinions using information and analysis.

Ethics can therefore be studied from the point of view of values, which are divided into three basics types: utilitarian values, moral values, and existential values [32]. The latter is the most important and most personal. There is no direct evidence that our lives have meaning, but it seems reasonable to just assume that it does. If you are studying mathematics, statistics, and science, then you are certainly motivated. You need to be considering what is good and bad, and also how you could justify a value judgment. From that, inner questioning derives what you will consider to be right and wrong, and thus your morality.

Excessive concentration on utilitarian values has become a major distraction for people in our society, and this has led to the problems of pollution, poverty, and the mismanagement of resources. Conservation efforts have struggled to make a case for preservation. Failure can sometimes be traced back to ill-fated attempts at appealing to a cost–benefit analysis of environmental issues. In fact, if the death of a species means profit for some group, then the price for delivery of a species increases as it becomes depleted, leading to increased harvesting. This works to snuff the species right out of existence. A heavy emphasis on utilitarian values is, by its very nature, unbalanced.

The program presented in this book is intended to take the anxiety out of learning mathematical concepts and tools. Problems should be worked to the point that it becomes clear to the reader that a new idea is now understood. Mastery of any specific topic is outside the scope of what this text can offer, and will require consulting works dedicated to such a goal. Most material presented is standard, albeit in an atypical order. Some proofs of important results are presented to reiterate the language of deductive reasoning, and the use of symbolic logic. However, in many cases, a sketch and example will be used to build the intuition needed later in the book.

Throughout the text the reader should be thinking critically. Indeed, there will certainly be places where the format fails, if only because we are all different people [30]. Be sure to imagine a better way to have presented a topic. This should instill confidence in your understanding and a willingness to create your own symbolisms to represent concepts. A well-rounded experience will involve four states of mind, according to educators. They are (1) Thinking-Watching, (2) Thinking-Doing, (3) Feeling-Watching, (4) Feeling-Doing. An emphasis on lectures must always begin with state (1). However, an enthusiastic lecturer will engage the students, creating a transition to state (2). If organized properly, lectures will end with the solution of a problem and a conclusion. Students can be asked about their suggested policies based on a computation. Furthermore, as is common with mathematical problems, formulating and solving in the abstract can have a very satisfying effect, resulting in state (3). At the end of the course, problems become projects, and students will have to do a series of steps to solve a single big problem. Once the topics of mathematical modeling and the scientific method are addressed, it is hoped that the reader will feel empowered. This will mean that state (4) has been reached. It may seem that feelings should play little or no part in any scientific investigation. However, it is now understood that feeling is connected with memory. The consequence is that feelings of achievement, which most consider to be good feelings, may result in a deeper understanding, and a greater motivation, in the student.[2] This, in itself, constitutes an advance in the fields of science and mathematics.

[2] Human motivation is an important area of study for which there are many theories [50].

Acknowledgments

The authors would like to thank our students, whose insights and questions over the years helped make this book possible. We also thank our colleagues at East Carolina University, especially Robert Bernhardt, John Crammer, Robert Joyner, Njina Randriampiry, Heather Ries, Catherine Rigsby, and Zach Robinson, and our colleague at the University of Windsor, Gerry McPhail, for their invaluable advice, support, and philosophical discourse in this and many other endeavors. The second author would like to thank Shondell Jones for his expertise at physical therapy, which allowed for timely completion of our book. We also express our thanks and gratitude to Tim Anderson, Lindsey Jones, Melissa Potter, and Amy Rose of Jones & Bartlett Learning, along with their excellent proofreading staff, for helping improve our book and bringing it to fruition.