Exercises

2.5.1 Answer the following questions related to geometric similarity and proportionality:

a. If water bottles are geometrically similar, how much more water will a bottle that is 30 cm tall hold than one that is 10 cm tall?

b. If air resistance is proportional to the surface area of a falling object at a given velocity, how much more air resistance will a sphere of diamater 5 cm encounter than one with a diameter of 1 cm?

c. If gymnasts have a constant density, then weight is proportional to volume. If we further assume that they are geometrically similar, how much less would a gymnast who is 5 ft. tall weigh than one who is 5.5 ft. tall?

d. If hearts are geometrically similar and the volume of blood pumped in one beat is proportional to the volume of the heart, how much more blood will a heart 4 cm wide pump in one beat than a heart that is 1 cm wide?

e. If the amount of heat lost by a submarine over a unit of time is proportional to its surface area, how much more heat will a submarine that is 50 ft. long lose over a given period of time than a scale model that is 10 ft. long?

f. If objects are geometrically similar and have a constant density, we saw in Example 2.5.2 that $A \propto W^{2/3}$ where A = Surface Area and W = Weight. If the weight of one such object is 5 times the weight of another, how much larger is the surface area?

g. Suppose that an ice cube melts so that at any point in time, the remaining cube is geometrically similar to the initial cube (i.e. before it started melting). At one point in time, the length is half the initial length. What fraction of the initial volume has melted?

2.5.2 Derive proportionality models in the following scenarios. Be sure to define variables in each model.

a. If some right triangles drawn on a page are geometrically similar, model the area of a triangle in terms of the length of its hypotenuse.

b. If some cubes in a bag are geometrically similar, model the volume of a cube in terms of the distance between opposite corners.

c. If some rocks in a bag are geometrically similar and have a constant density, model the surface area of a rock in terms of its weight.

d. If the sides of certain hollow aluminum tubes are 1 mm thick and the outer surface of the tubes are geometrically similar, model the volume of aluminum used to make a tube in terms of its length.

e. If some cargo airplanes are geometrically similar, the lift capacity of the wings is proportional to the surface area of the wings, and the cargo capacity of an airplane is directly proportional to the lift capacity of its wings, model the cargo capacity of an airplane in terms of its wing span.

f. If some books on a shelf are geometric similar with a constant density and the thickness of a book is directly proportional to the number of pages, model the weight of a book in terms of its number of pages.

g. If some flowers are geometrically similar and the height of a flower is directly proportional to the number of leaves on the flower, model the surface area of a flower's bloom in terms of the number of leaves.

2.5.3 When a pineapple is peeled, the resulting edible fruit has approximately the shape of a right–circular cylinder. Let d denote the diamater of this cylinder.

a. Assuming pineapples are geometrically similar and have a constant density, derive a model for the weight of a pineapple's edible fruit in terms of d.

b. Consider 6 "small" pineapples each of diamater 5 in, and 6 "large" pineapples each of diamater 5.5 in. In terms of a percentage, how much more total fruit do the "large" pineapples have than the "small" pineapples?

2.5.4 An asparagus spear has approximately the shape of a right–circular cone. Let d and V denote the diameter of the base and volume of this cone, respectively. Assume that all asparagus spears are geometrically similar. Some parts of a spear are tender and very edible, but some are tough and not edible. Typically spears with a larger diameter are tougher than those with a smaller diameter. In parts a–c, model the edible volume of a spear in terms of d under the given assumption.

a. The edible volume is some fraction (or proportion) of V.

b. The edible volume is proportional to V/d.

c. The edible volume is proportional to $V/d^{3.5}$.

d. Which, if any, of the above assumptions seems most reasonable to you? Explain.

e. Consider the assumption in part c. Suppose you have the option of order 10 "small" spears each of diamater 0.3 in, or 10 "large" spears each of diamater 0.4 in. Which option will have more edible volume? Which option would you choose?

2.5.5 Answer the following questions regarding Example 2.5.2:

a. We showed that $A_s \approx 1.26A_L$. Besides the fact that this number has been rounded to two decimal places, explain why it would not be appropriate to claim that the surface area of the small potatoes is exactly 26% larger than the large potatoes.

b. We assumed that potatoes are geometrically similar. Describe how you might collect data to test the reasonableness of this assumption and the validitity of the model derived in this example.

2.5.6 Suppose it takes 0.75 ounces of sunscreen to cover all exposed areas of a 3-foot tall child. Estimate the amount of sunscreen it would take to cover all exposed areas of a 6-foot tall man. List all assumptions you make.

2.5.7 Consider a set of geometrically similar objects.

1. Derive a model for the ratio $\frac{\text{Surface Area}}{\text{Length}}$ in terms of Length. (Hint: Use length as the characteristic dimension and model surface area in terms of length.)
2. Derive a model for the ratio $\frac{\text{Volume}}{\text{Length}}$ in terms of Length.

2.5.8 Ace manufacturing company owns a warehouse in the shape of a rectangular box that measures 100 ft. wide by 200 ft. long by 10 ft. high. It has a furnace with an output of 500,000 BTU's. They want to build a larger warehouse measuring 200 ft. by 400 ft. by 20 ft. and are trying to determine what size of furnace needs to be installed.

a. If they assume that the size of the furnace is proportional to the volume of the building, what size furnace should be installed?
b. If they assume that the size of the furnace is proportional to the total surface area of the building (including the floor), what size furnace should be installed?
c. If they assume that the size of the furnace is proportional to the surface area of the walls and roof *only* (meaning the floor is not a factor), what size furnace should be installed?

2.5.9 Table 2.9 gives the overall length (inches) and weight (pounds) of several male black bears. If we assume male black bears are geometrically similar, then we would expect that Weight $\propto$ Length^3. Use this data in the table to determine if geometric similarity is a reasonable assumption.

Length	138	166	180	129.5	150	132	148	140	137	149
Weight	60	155	220	105	110	75	105	90	75	115
Length	102	173	104.5	138	144.5	164	129	158	150	142
Weight	35	220	33	90	80	180	77	120	100	100

Table 2.9

2.5.10 A rowing shell is a slim, needle–like boat built for speed that is powered by one, two, four, or eight rowers. We want to build a model that predicts the boat's speed, v, in terms of the number of rowers, r. Table 2.10 contains data on boat length and winning race times at four world championships (reprinted with permission from Science News, copyright 2008).

		Time for 2,000 m (min)			
Rowers, r	**Length, l**	**I**	**II**	**III**	**IV**
8	18.28	5.87	5.92	5.82	5.73
4	11.75	6.33	6.42	6.48	6.13
2	9.76	6.87	6.92	6.95	6.77
1	7.93	7.16	7.25	7.28	7.17

Table 2.10

Consider the following assumptions:

1. $l \propto r^{1/3}$ where l is the length of the boat.
2. The only drag slowing down the boat is from the water, which is proportional to Sv^2 where S is the wetted surface area of the boat.
3. The boats travel at a constant speed. This means that the force applied by the rowers equals the force of drag.
4. All boats are geometrically similar with regard to S.
5. The power available to the boat is proportional to r. (Power = Force × Speed where Force is the force applied by the rowers)

Is the first assumption reasonable? Test it with the given data. Build a model that predicts v in terms of r and test it with the given data. Here are a few suggestions:

1. Start with the relationship Power = Force × Speed and substitute the proportionality relationships from the other assumptions.
2. Remember, velocity = distance/time.
3. Use the average time for the 2,000 m race to calculate the velocity.

2.5.11 Consider a raindrop falling from a cloud. Ignoring any affects of wind, it is influenced by two forces, weight due to gravity and air resistance. At some point, the raindrop reaches a terminal velocity, v_t, where these two forces are equal.

Consider the following assumptions:

1. All raindrops are geometrically similar.
2. All raindrops have the same density.
3. Air resistance is proportional to the product of its surface area and the square of its speed.

Use these assumptions to model the terminal velocity of a raindrop in terms of its weight.

For Further Reading

- For more examples of modeling with proportionality, see GIORDANO/WEIR/FOX, *A First Course in Mathematical Modeling*, 3e, 2003, pages 95 – 96, ©Brooks/Cole, a part of Cengage Learning, Inc.
- For examples of modeling biological systems with proportionality, see A.J. Clark, *Comparative Physiology of the Heart*, Macmillian, 1927.