Exercises

4.6.1 In this section we argued that the initial number of cases may be under-reported. If this was the case and the number of new cases in week 1 was constant, then the model predicts that the epidemic may not be as bad as originally thought. One could argue that the number of new cases in week 1 could also be under-reported. Use the model to investigate the severity of the epidemic if the number of new cases in week 1 is higher than 5 but the initial number of cases is 3. What happens if both numbers are higher than originally suspected?

4.6.2 In the original model we assumed that the population is constant. Now let's relax this assumption. Suppose that 25 people move into the community each week starting in week 1. Assume that each of these people is susceptible.

a. Modify the worksheet to incorporate this influx of people and describe what happens to the spread of the flu over a 100 week period (use $\alpha = 0.00167$, $\gamma = 0.5$ and $I_0 = 3$).

b. What happens if 100 new people move in each week?

4.6.3 Suppose the population is a constant 1,000, that initially 50 people have the flu ($I_0 = 50$), $\alpha = 0.00167$, and that $\gamma = 0.5$. To try to decrease the severity of the empidemic, the community quarantines some of those with the flu. One way to model quarantining is to simply modify the transmission coefficient. For instance, if 25% of those with the flu are quarantined, then only 75% of the interactions between those infected and those susceptible are capable of producing an infected person. Therefore, the number of newly infected people is given by

$$(0.75\,\alpha)\, S_n I_n$$

Thus the new "effective" transmission coefficient is $(0.75\,\alpha)$.

a. Add a cell to the worksheet "**Epidemics**" to hold the new parameter "Proportion Quarantined," set it equal to 0.25, and modify the model to incorporate this parameter. Describe what effect quarantining 25% has over not quarantining.

b. Add a scroll bar that allows the user to vary the proportion quarantined between 0 and 1. Describe what happens as the proportion quarantined changes. What happens if it equals 0? What if it equals 1?

c. Add a scroll bar that allows the user to vary the number of initial cases, I_0, between 0 and 1000.

d. Find a value of Proportion Quarantined that prevents an epidemic from occurring (i.e. the number of infectives never increases) regardless of the value of I_0. Estimate the smallest such value of Proportion Quarantined.

4.6.4 Consider a disease such as the common cold where a person is *not* immune once they are "healed." Once healed, a person becomes susceptible again. Such as disease could be modeled with an SIS model as illustrated in Figure 4.33.

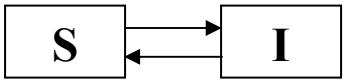

Figure 4.33

a. Devise a set of equations for an SIS model. (**Hint:** There is no "Removed" category. Infected people are "healed" and are immediately added to the susceptible category.)

b. Implement your model in an Excel worksheet to describe the spread of the common cold through a population of 1,000 where initially 4 people have the cold and assuming that the cold lasts an average of 2 weeks (use $\alpha = 0.00167$). What do you observe?

For Further Reading

- For an extremely well–written treatise on discrete dynamical systems, see Sandefur, James T, *Discrete Dynamical Systems Theory and Applications*, Clarendon Press, 1990.
- For additional examples of discrete dynamical systems, see Meerschaert, Mark M., *Mathematical Modeling*, Second ed., Academic Press, 1999, pg. 141 – 152. Also see the bibliography given on page 152.
- For examples of discrete dyanmical systems applied to compound interest and mortgage payments, see Tung, Ka–Kit, *Topics in Mathematical Modeling*, Princeton University Press, 2007, pg. 54 – 67.
- For additional information on epidemic models, see Allman, Elizabeth Spencer and John A. Rhodes, *Mathematical Models in Biology: An Introduction*, Cambridge University Press, 2004.
- For information on analyzing discrete dynamical systems with eigenvalue and eigenvector methods, see Lay, David C., *Linear Algebra and its Applications*, Third ed., Addison Wesley, 2003, pg. 342 – 353.
- For a much different approach to modeling dynamical systems, see Hannon, Bruce and Matthias Ruth, *Dynamic Modeling*, Springer, 2001.