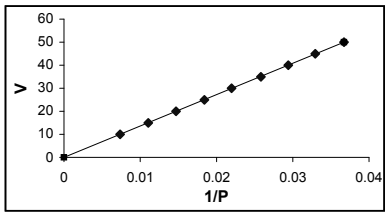

Figure 2.10

Exercises

2.2.1 For each of the data sets below, determine if it is reasonable to assume that y is proportional to x. If it is, approximate the constant of proportionality. If it is not, describe why this assumption is not reasonable.

a.

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7
y	1	1.21	1.44	1.69	1.96	2.25	2.56	2.89

b.

x	1	5	7	2	10	12	3	6
y	0.79	10.89	14.37	5.75	23.36	26.29	3.76	16.12

c.

x	2	6	9	15	7	25	39	4
y	26	20	18	26	6	19	20	13

2.2.2 Suppose you drive your car on a perfectly flat road at a constant speed with no wind. In this case, the amount of fuel, y, (in gallons) needed is directly proportional to the distance traveled, x (in miles).

a. If the distance traveled increases, what can we say about the amount of fuel needed?

b. If the relationship is given by $y = 0.04x$ and x increases by 50 miles, how much does y increase?

c. Now, suppose it takes 12 gallons of fuel to travel 282 miles. Find the constant of proportionality.

d. In words, describe the meaning of this constant of proportionality.

2.2.3 A variable y is said to be *inversely* proportional to x if there exists a constant c such that $y = \frac{c}{x}$.

a. If y is inversely proportional to x, sketch what a graph of y vs. x would look like. What would a graph of y vs. $1/x$ look like?

b. If y is inversely proportional to x, show that x is inversely proportional to y.

c. If y is inversely proportional to x and x increases, what happens to the value of y?

2.2.4 For the data set below, determine if it is reasonable to assume that y is inversely proportional to x. If it is, approximate the constant of proportionality. If it is not, describe why this assumption is not reasonable.

x	1	1.2	1.4	1.6	1.8	2.0	2.2	2.4
y	6.85	6.21	4.24	4.32	3.92	3.18	2.93	2.96

2.2.5 Newton's Law of Universal Gravitation states that the force of attraction F between two objects with masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance d between them. In mathematical notation,

$$F = k\frac{m_1 m_2}{d^2}$$

For two given objects, if m_1 and m_2 are constant, we may combine them with the constant of proportionality k to describe the relationship by

$$F = \frac{C}{d^2}$$

where C is a constant. If one of the two objects is a planet, the distance d is the distance from the center of the planet to the second object.

a. The radius of the Earth is approximately 4,000 miles. A satellite weighs 15 tons on the surface of the Earth (i.e. the force of attraction between the Earth and the satellite is 15 tons at the surface of the Earth). Use this information to calculate the constant of proportionality C.
b. Find the weight of the satellite (in tons) at an altitude of 500 miles above the surface of the Earth.
c. Use Excel to graph the weight of the satellite vs. altitude for values of altitude between 0 and 4,000 miles (ignore the affects of all other celestial bodies like the Moon).

2.2.6 Determine which of the following models "best" fits the data below by transforming the data appropriately and fitting a straight line to the transformed data. Find the constant of proportionality for this model. Explain why your choice is the "best" model.

$$y \propto x, \quad y \propto \frac{1}{x}, \quad y \propto x^2, \quad y \propto \sqrt{x}, \quad y \propto \frac{1}{x^3}$$

x	0.5	0.7	0.9	1.2	1.5
y	7.8	3.5	2.2	0.85	0.36

2.2.7 In Figures 2.5 and 2.10 we drew the straight line fit to the data (or the transformed data) so that the line went through the origin.

a. If $y \propto x$, explain why the straight line fit to the data *should* go through the origin.
b. If $y \propto u(x)$, where $u(x)$ is some function of x, explain why the straight line fit to graph of the transformed data (y vs $u(x)$) *should* go through the origin.

2.2.8 A common assumption regarding the metabolism of a drug in the blood stream is that the rate of change of the concentration of the drug is proportional to the concentration. That is, if A is the concentration of the drug, then $dA/dt \propto A$. The data below shows the concentration of a certain drug in a person's blood stream, A, taken t hours after the drug was administered.

t	0	1.2	2.4	3.5	4.8	6.4	9.4	10.5	12.4	15.6	18.8
A	10.2	8.6	7.3	6.2	5.2	4.2	2.7	2.3	1.8	1.2	0.7

a. The change in the concentration at hour t is approximately

$$\frac{dA}{dt} \approx \frac{A(t+\Delta t) - A(t)}{\Delta t}$$

where Δt is some small change in t. Use the data to approximate dA/dt for the given values of t.

b. Graph dA/dt vs A. Does it appear reasonable to assume that $dA/dt \propto A$? If so, approximately what is the constant of proportionality?

2.2.9 The kinetic energy (in J) of a moving object is $K_E = 0.5mv^2$ where m is the mass of the object (in kg) and v is the velocity (in m/s). Suppose a researcher measures the kinetic energy of an object at different velocities as shown below. Use the data to estimate the mass of the object.

v (m/s)	1.3	2.5	3.4	5.2	8.4	9.1	10.1	10.9
$\mathbf{K_E(\times 10^{-5} J)}$	0.132	0.502	0.891	2.22	5.63	6.41	8.15	9.84

2.2.10 Fit a model of the form $y \propto sin(3x+1)/\left(x^2-2\right)$ to the data below. Give the constant of proportionality

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y	-2.42	-2.54	-2.39	-2.14	-1.64	-0.98	-0.15	0.95	2.15	3.84

2.2.11 Prove each of the following properties of proportionality:

a. If $ab \propto ac$ and $a \neq 0$, then $b \propto c$.
b. If $a^m \propto ac$ and $a \neq 0$, then $a^{m-1} \propto c$.
c. If $a \propto c^m$, then $a^{1/m} \propto c$.
d. If $y \propto (x/z)$, $x \propto h^n$, and $z \propto h^m$ where n, $m > 0$, then $y \propto h^{n-m}$.

2.2.12 A snow-cone seller at a county fair wants to model the number of cones he will sell, C, in terms of the daily attendance a, the temperature T, the price p, and the number of other food vendors n. He makes the following assumptions:

a. C is directly proportional to a and the difference between T and 85 °F
b. C is inversely proportional to p and n

Derive a model for C consistent with these assumptions. For what values of T is this model valid?