Exercises

4.3.1 Table 4.4 contains data on the population of foxes in a forest over a period of several years. Fit a discrete logistic equation to the data. How well does the model fit the data?

n	0	1	2	3	4	5	6	7	8	9	10
a_n	50	85	110	130	175	200	215	221	228	232	234

Table 4.4

4.3.2 In this exercise we will fit a discrete logistic equation to the data in Table 4.2 using the least–squares criterion. This criterion says, informally, that when predicting the values of data $\{a_n \mid n = 1, \ldots, m\}$, the number

$$\sum_{i=1}^{m} (a_n - \text{Predicted})^2$$

(called the sum of squares) should be as small as possible. With a carrying capacity of 621, use a scroll bar to find a value of b in the discrete logistic equation that minimizes the sum of squares. Does this give the same model as the one found in Example 4.3.1?

Here are some suggestions:

1. Modify the worksheet **Bacteria** by adding a column titled "$(a_n - \text{Predicted})^2$." Sum the values in this column to calculate the sum of squares.
2. Create a cell to hold the value of b. Reference this cell in your formula for the predicted values.
3. Add a scroll bar with a min and max of 0 and 1,000. Create a formula for the value of b equal to the scroll bar linked cell divided by 500,000.
4. Move the slider back and forth to find a value of b that minimizes the sum of squares.

4.3.3 Suppose we estimate that a forest can support a population of 10,000 deer and that the population of deer is described by the model $a_{n+1} = a_n + 0.00006(10000 - a_n)\, a_n$ where a_n is the population at the end of year n.

1. Suppose that we let hunters kill 700 deer at the end of each year. Write a model to describe this situation and analyze the long–term behavior of the population for different initial populations.
2. Suppose we start with a population of 9,000 deer and we allow hunters to kill m deer at the end of each year. Analyze the long–term behavior of the population for different values of m. What is the maximum value of m for which the population survives in the long–term?
3. Suppose we start with a population of 9,000 deer and we allow hunters to kill a certain proportion of the population (such as 0.25) at the end of each year. Write a model to describe this situation and analyze the long–term behavior of the population for different values of the proportion.