Exercises

5.3.1 Investigate sensitivity of the fox–wolf model (5.5) to the carrying capacities. That is, change the carrying capacities a small amount and analyze the resulting model. Does your final conclusion change?

5.3.2 Consider the predator–prey system modeled in Example 5.3.4. Use a scroll bar to investigate the sensitivity of the model to the parameter c_2.

a. What happens to the equilibrium value as c_2 changes?
b. What happens to the variation within each population as c_2 changes?
c. If shelters were built to protect the rabbits from the foxes, what does the model predict might happen to the populations? Would it increase the size of the rabbit population?

5.3.3 Suppose two populations, call them x and y, are described by the quadratic population model (5.5) and (5.6). Further suppose that in the absense of competition both populations grow. For each of the following scenarios, give possible values of the parameters a_1 through c_2.

a. For both populations, there is no competition within or between species.
b. For both populations, there is no competition within species and competition between species decreases the rate of growth of both species.
c. For both populations, competition within species decreases the rate of growth and competition between species increases the rate of growth.
d. For population x, competition within species increases the rate of growth and competition between speciese decreases the rate of growth. For population y, the opposite is true.
e. For both populations, competition decreases the rate of growth, but competition between species is much greater than competition within species.

5.3.4 The motions of a certain pendulum are described by the system of differential equations:

$$\frac{dx}{dt} = y, \ \frac{dy}{dt} = -5\sin x - \frac{9}{13}y$$

where $x = \theta$, the angle between the rod and the downward vertical direction, and $y = \frac{d\theta}{dt}$, the speed at which the angle changes. Describe the equilibrium values for this system.

5.3.5 Suppose the populations of two species are described by the following system of differential equations:

$$\begin{aligned}\frac{dx}{dt} &= -x + y\\ \frac{dy}{dt} &= -y + x.\end{aligned}$$

a. According to this model, can either species survive without the other? Explain why or why not.

b. Find the equilibrium points of this system.

c. Describe the long-term behavior of the system for different initial populations.

d. Would you describe this system as being sensitive to the initial populations? Explain why or why not.

5.3.6 Consider a system of linear differential equations of the form

$$\frac{dx}{dt} = ax + by$$
$$\frac{dy}{dt} = cx + dy.$$

a. Show that the origin (0, 0) is an equilibrium point for any values of a, b, c, and d.

b. For each of the following values of the parameters (a, b, c, d), graphically determine if the origin is an attracting equilibrium point (meaning trajectories always move toward the orign) or repelling (meaning trajectories always move away from the origin).

i. $(-2, -5, 1, 4)$ ii. $(7, -1, 3, 3)$
iii. $(-3, -2, -1, -1)$ iv. $(3, 1, -2, 1)$
v. $(-3, -9, 2, 3)$

5.3.7 A predator–prey model that takes into account harvesting (i.e., hunting) of the two species is

$$\frac{dx}{dt} = a_1x - b_1xy - cx$$
$$\frac{dy}{dt} = -a_2y + b_2xy - cy$$

Where $x(t)$ and $y(t)$ are the populations of the prey and predator species, respectively. All parameters are assumed to be positive. Assuming that $x \neq 0 \neq y$, find a formula for the equilibrium value in terms of the parameters.

5.3.8 Consider the predator-prey system in Example 5.3.4. In this model we used constant values of a_1 and a_2 (the numbers -0.08 and 0.04). In the terminology of Chapter 4, these are the fox death and rabbit birth factors, respectively. Now suppose these factors change throughout the year. Suppose that during the "summer", these factors are -0.08 and 0.04, respectively, but during the "winter" they are -0.1 and 0.01, respectively.

a. Use Euler's method to estimate the populations from month 0 to month 250 using $h = 1$. Create one graph of the fox population vs month and another graph for the rabbit population. Let the "winter" occur from month 0 through 5, then 12 through 17, and so on.

b. Compare the populations when the factors are constant to the populations when the factors vary. Do the varying factors cause the ranges of the populations to increase or decrease?