Exercises

6.2.1 Modify the formulas in the worksheet **Coins** so there are a total of 1,000 trials. Press **F9** several times. Compare the variability of the value of the probability in 200 trials as compared to 1,000 trials.

6.2.2 Modify the worksheet **Coins** to approximate the probability that the number of Tails is between 4 and 7, inclusive. (**Suggestion:** Add a column next to Total Tails to indicate whether or not the trial is a success (meaning the number of Tails is between 4 and 7). The formulas in this column may look similar to that in Figure 6.11. Modify the formula to count the number of successes.)

	M
1	**Success?**
2	=IF(AND(L2>=4,L2<=7),1,0)

Figure 6.11

6.2.3 Suppose you roll a fair four-sided die three times. Design a spreadsheet to estimate the probability that

a. the sum of the three rolls is at least 7,
b. at least one of the rolls is greater than 2, and
c. the first roll or the third roll is even.

(**Suggestion:** To determine if a trial is a success for probabilities b. and c., use the **OR** function which works much like the **AND** function in Figure 6.11.)

6.2.4 Consider the area under a curve model in Example 6.2.2.

a. Increase the value of h in the simulation. How does this affect the quality of the estimation of the area?
b. Increase the number of points selected inside the rectangle. How does this affect the estimate?
c. Create a graph of the curve $y = \sqrt{1 - x^2}$ and the rectangle along with a dot for each randomly selected point, similar to Figure 6.12.

6.2.5 Design a simulation to estimate the value of $\int_0^{0.5} xe^x \sin(50x)\, dx + 1$.

6.2.6 A square is constructed such that the length of a side is randomly chosen between 2 and 3 inches (not necessarily an integer). Design a simulation to estimate the probability that the area of the square is between 5 and 6 in^2.

6.2.7 Consider the following "game:"

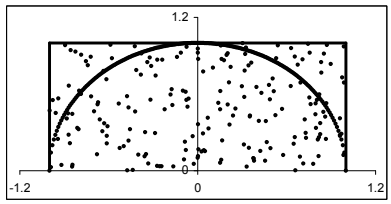

Figure 6.12

In each round, two numbers, call them N_1 and N_2 are chosen. In the first round, $N_1 = 1$, in the second, $N_1 = 2$, and so on until round 11 when $N_1 = 1$. This pattern repeats until the game is over. In each round, the number N_2 is a randomly chosen integer between 1 and 10, inclusive. The game is over when $N_1 = N_2$.

a. Simulate this game to estimate the average number of rounds in the game.

b. Generalize the simulation by replacing the number 10 with any positive integer N. Allow the user to enter the value of N.

6.2.8 Consider the following "game:"

You choose an integer y between 1 and 20, inclusive. Then the dealer randomly chooses a number x, not necessarily an integer, between 0 and 10. The amount of money you "win" is given by $z = -x^2 + 8x - y^2 + 24y - 175$ (if $z > 0$, you get that many dollars, if $z < 0$, you pay that many dollars).

Simulate this game and determine the value of y you should choose to maximize the average winnings from many plays of the game.

6.2.9 A car dealership is sponsoring a contest where the grand prize is a new car. Contests are to gather tickets which contain the letter "C," "A," or "R" from participating merchants. To win, one must obtain all three letters. If 55% of the tickets contain a "C", 44% contain an "A", and 1% contain an "R," design a simulation to approximate the expected number of tickets you must gather to win the car.

Hints:

1. Keep track of the number of each letter you have collected with each ticket.
2. Remember that you win only after you have collected at least 1 of each letter.
3. Make sure you simulate collecting enough tickets in each trial so that you are almost certain to win in each trial.
4. To calculate the letter on each ticket, consider using formulas as in Figure 6.13.

	B	C
3	**Random**	
4	**Number**	**Letter**
5	=RAND()	=IF(B5<0.55,"C",IF(B5<0.99,"A","R"))

Figure 6.13

6.3 Three Famous Problems

In this section we show how simulations can be used to approximate the solutions to three famous problems in elementary probability: the *Monty Hall Problem*, the *Birthday Problem*. and *Buffon's Needle Problem*.

The Monty Hall Problem

In the famous game show *Let's Make a Deal*, hosted by Monty Hall, one of the games required a contestant to choose one of three doors. Behind one of the doors was a prize (like money or a car), and behind the other two doors were dummy prizes (like a donkey). Once the choice was made, Monty Hall would open up one of the un-chosen doors revealing one of the dummy prizes. The contestant was then given a choice to either switch to the remaining unopened door or keep the door that was already chosen. The contestant would get whatever "prize" was behind the door.

To illustrate this game, consider the scenario in Figure 6.14 where the real prize is behind door 2 (unbeknownst to the constentant) and the contestant chooses door 1. Monty would then open door 3 revealing a dummy prize. The contestant then had to decide whether to switch to door 2 (and consequently win the real prize) or don't switch (and consequently not win the real prize).

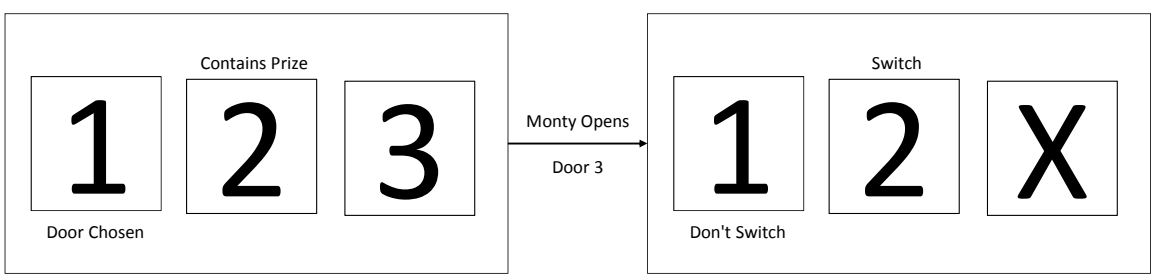

Figure 6.14

The dillemma facing the contestant is the decision of whether to switch or not. To help make this decision, we will use a simulation to estimate the following two probabilities:

1. Probability of winning if switching and